Chapter 5: Factorization of Algebraic Expressions

Exercise 5.1

Q.1) Factorize x^3+x-3x^2-3.

Soln:- Given that, x^3+x-3x^2-3.

$$x^3+x-3x^2-3 = x^3-3x^2+x-3$$
$$= x^2(x-3)+1(x-3)$$
$$= (x^2+1)(x-3)$$

Thus, $\boxed{(x^3+x-3x^2-3) = (x^2+1)(x-3)}$

Q.2) Factorize $a(a+b)^3-3a^2b(a+b)$.

Soln:- $a(a+b)^3-3a^2b(a+b) = a(a+b)[(a+b)^2-3ab]$

$$= a(a+b)[a^2+2ab+b^2-3ab]$$

$\because (a+b)^2 = a^2+b^2+2ab$

$$= a(a+b)[a^2+b^2-ab]$$

Thus, $\boxed{a(a+b)^3-3a^2b(a+b) = a(a+b)(a^2+b^2-ab)}$

Q.3.) Factorize $x(x^3-y^3)+3xy(x-y)$.

Soln:- Given that,

$$x(x^3-y^3)+3xy(x-y) = x(x-y)(x^2+xy+y^2)+3xy(x-y)$$

$\because (x^3-y^3) = (x-y)(x^2+xy+y^2)$

$$= x(x-y)[x^2+xy+y^2+3y]$$

$$= x(x-y)(x^2+xy+y^2+3y)$$

Thus, $\boxed{x(x^3-y^3)+3xy(x-y) = x(x-y)(x^2+xy+y^2+3y)}$

Q.4.) Factorize $a^2x^2+(a^2x^2+1)x+a$.

Soln:- $a^2x^2+(ax^2+1)x+a$

Here, $a^2x^2+(ax^2+1)x+a = a^2x^2+a+x(ax^2+1)$

$$= a(ax^2+1)+x(ax^2+1)$$

$$= (a+x)(ax^2+1)$$

Thus, $\boxed{a^2x^2+(ax^2+1)x+a = (a+x)(ax^2+1)}$

Q.5.) Factorize $x^2+y-xy-x$.

Soln:- Here, $x^2+y-xy-x$

$\Rightarrow$ Now, $x^2+y-xy-x = x(x)-xy+y-x$

$$= x(x-y)-(x-y)$$

$$= (x-y)(x-1)$$

Thus, $\boxed{(x^2+y-xy-x) = (x-y)(x-1)}$

Q.6) Factorize $x^3-2x^2y+3xy^2-6y^3$.

Soln:- Here, $x^3-2x^2y+3xy^2-6y^3$

$= x^2(x-2y)+3y^2(x-2y)$

$= (x-2y)(x^2+3y^2)$

Thus, $\boxed{(x^3-2x^2y+3xy^2-6y^3) = (x-2y)(x^2+3y^2)}$

Q.7) Factorize $6ab-b^2+12ac-2bc$.

Soln:- $6ab-b^2+12ac-2bc$

$= 6ab+12ac-b^2-2bc$

$= 6a(b+2c)-b(b+2c)$

$= (6a-b)(b+2c)$

Thus, $\boxed{(6ab-b^2+12ac-2bc) = (6a-b)(b+2c)}$

Q.8) Factorize $(x^2+1/x^2)-4(x+1/x)+6$.

Soln:- Here, $(x^2+1/x^2)-4(x+1/x)+6$

$= x^2+1/x^2-4x-4/x+6$

$= x^2+1/x^2+4+2-4/x-4x \quad \because 6=4+2$

$= (x)^2+(\frac{1}{x})^2+(-2)^2+2x(1/x)+2(1/x)(-2)+2(-2)x$

we have, $a^2+b^2+c^2+2ab+2bc+2ac=(a+b+c)^2$

Thus,

$= [x+1/x+(-2)]^2$

Thus, $\boxed{\left(x^2+1/x^2\right)-4\left(x+1/x\right)+6 = \left(x+1/x-2\right)^2}$

Q.9.) factorize $x(x-2)(x-4)+4x-8$.

Soln:- Given that,

$$x(x-2)(x+4)+(4x-8) = x(x-2)(x-4)+4(x-2)$$
$$= (x-2)[x(x-4)+4]$$
$$= (x-2)(x^2-4x+4)$$
$$= (x-2)(x-2)^2$$

Thus,

$$\boxed{x(x-2)(x-4)+4x-8 = (x-2)^3}$$

Q.10.) factorize $(x+2)(x^2+25)-10x^2-20x$.

Soln:- Given that,

$$= (x+2)(x^2+25)-10x^2-20x$$
$$= (x+2)(x^2+25)-10x(x+2)$$
$$= (x+2)[(x^2+25)-10x]$$
$$= (x+2)[x^2-10x+25] \quad \because a^2-2ab+b^2 = (a-b)^2$$
$$= (x+2)(x-5)^2$$

Thus, $\boxed{(x+2)(x^2+25)-10x^2-20x = (x+2)(x-5)^2}$

Q.11.) Factorize $2a^2+2\sqrt{6}ab+3b^2$.

Soln:- Given that,

$2a^2+2\sqrt{6}ab+3b^2$

$=(\sqrt{2}a)^2+2\times(\sqrt{2}a)(\sqrt{3}b)+(\sqrt{3}b)^2$

But, we have $(a+b)^2=a^2+b^2+2ab$

Here, $\boxed{2a^2+2\sqrt{6}ab+3b^2=(\sqrt{2}a+\sqrt{3}b)^2}$

Q.12.) Factorize $(a-b+c)^2+(b-c+a)^2+2(a-b+c)(b-c+a)$.

Soln:-

Given, $(a-b+c)^2+(b-c+a)^2+2(a-b+c)(b-c+a)$

we have, $a^2+b^2+2ab=(a+b)^2$

Here, $\Rightarrow (a-b+c)^2+(b-c+a)^2+2(a-b+c)(b-c+a)$

$=(a-b+c+b-c+a)^2$

$=(2a)^2$

$=4a^2$

Thus, $\boxed{(a-b+c)^2+(b-c+a)^2+2(a-b+c)(b-c+a)=4a^2}$

Q.13) Factorize ~~4(x~~ $a^2+b^2+2(ab+bc+ca)$.

Soln:- Given that, $a^2+b^2+2(ab+bc+ca)$

$=(a^2+b^2+2ab)+2(bc+ca)$

$=(a+b)^2+2c(a+b) \quad \because (a^2+b^2+2ab)=(a+b)^2$

$=(a+b)^2+2c(a+b)$

$=(a+b)[a+b+2c]$

Thus, $\boxed{a^2+b^2+2(ab+bc+ca)=(a+b)(a+b+2c)}$

Q.14) Factorize $4(x-y)^2-12(x-y)(x+y)+9(x+y)^2$

Soln:- Given that,

$$4(x-y)^2-12(x-y)(x+y)+9(x+y)^2 \quad \text{--- (1)}$$

put $(x-y)=p$ and $(x+y)=q$ in equn (1).

$$(1) \Rightarrow = 4p^2-12pq+9q^2$$

$$= 4p^2-6pq-6pq+9q^2$$

$$= 2p(2p-3q)-3q(2p-3q)$$

$$= (2p-3q)(2p-3q)$$

$$= (2p-3q)^2$$

But, $p=x-y$ & $q=x+y$

Thus, $4(x-y)^2-12(x-y)(x+y)+9(x+y)^2$

$$= [2(x-y)-3(x+y)]^2$$

$$= [2x-2y-3x-3y]^2$$

$$= (-x-5y)^2$$

$$= [(-1)(x+5y)]^2$$

$$= (x+5y)^2$$

Thus, $\boxed{4(x-y)^2-12(x-y)(x+y)+9(x+y)^2=(x+5y)^2}$

Q.15) Factorize $a^2-b^2+2bc-c^2$.

Soln:- Given that,

$$a^2-b^2+2bc-c^2$$

$$=(a^2-b^2+2bc-c^2)$$

$$=a^2-(b^2-2bc+c^2) \qquad \because (a-b)^2=a^2+b^2-2ab$$

$$=a^2-(b-c)^2$$

And $a^2-b^2=(a-b)(a+b)$

$$\Rightarrow =(a-b+c)(a+b-c)$$

$\Rightarrow$ Thus, $\boxed{(a^2-b^2+2bc-c^2)=(a-b+c)(a+b-c)}$

Q.16) factorize $a^2+2ab+b^2-c^2$.

Soln:- Here, $a^2+2ab+b^2-c^2$.

$$\Rightarrow (a^2+2ab+b^2)-c^2$$

$$\Rightarrow (a+b)^2-c^2 \qquad \because (a+b)^2=a^2+2ab+b^2$$

$$\Rightarrow (a+b+c)(a+b-c) \qquad \because (a^2-b^2)=(a+b)(a-b)$$

Thus, $\boxed{a^2+2ab+b^2-c^2=(a+b+c)(a+b-c)}$

Exercise 5.2

Q.1) Factorize each of the following expressions:

i) p^3+27

$\rightarrow$ Given, $p^3+27 = p^3+3^3$

$\because a^3+b^3 = (a+b)(a^2-ab+b^2)$

$$\boxed{p^3+27 = (p+3)(p^2-3p+9)}$$

ii) y^3+125

$\rightarrow$ Given, $y^3+125 = y^3+(5)^3$

$\because (a^3+b^3) = (a+b)(a^2-ab+b^2)$

$\Rightarrow y^3+125 = (y+5)(y^2-5y+5^2)$

$$\boxed{y^3+125 = (y+5)(y^2-5y+25)}$$

iii) $1-27a^3$

$\rightarrow$ Given, $1-27a^3 = (1)^3-(3a)^3$

$\because (a^3-b^3) = (a-b)(a^2+ab+b^2)$

$$\Rightarrow \boxed{1-27a^3 = (1-3a)(1+3a+9a^2)}$$

iv) $8x^3y^3+27a^3$

$\rightarrow$ Given, $8x^3y^3+27a^3 = (2xy)^3+(3a)^3$

$\because (a^3+b^3) = (a+b)(a^2-ab+b^2)$

$$\Rightarrow \boxed{8x^3y^3+27a^3 = (2xy+3a)(4x^2y^2-6xya+9a^2)}$$

v) $64a^3-b^3$

→ Given, $64a^3-b^3 = (4a)^3-(b)^3$

$\because (a^3-b^3)=(a-b)\ (a^2+ab+b^2)$

$\Rightarrow 64a^3-b^3 = \cancel{(64a-b)}\ \cancel{(64a^2}$

$$\boxed{64a^3-b^3 = (4a-b)\ (16a^2+4ab+b^2)}$$

vi) $x^3/216-8y^3$

→ Given, $x^3/216-8y^3$

$x^3/216-8y^3 = \left(\frac{x}{6}\right)^3-(2y)^3$

But, we have $(a^3-b^3)=(a-b)\,(a^2+ab+b^2)$

$\Rightarrow \left(\frac{x}{6}\right)^3-(2y)^3 = \left(\frac{x}{6}-2y\right)\left[\left(\frac{x}{6}\right)^2+\left(\frac{x}{6}\right)(2y)+(2y)^2\right]$

$= \left(\frac{x}{6}-2y\right)\left[\frac{x^2}{36}+\frac{xy}{3}+4y^2\right]$

$$\therefore \boxed{\left(\frac{x^3}{216}-8y^3\right) = \left(\frac{x}{6}-2y\right)\left(\frac{x^2}{36}+\frac{xy}{3}+4y^2\right)}$$

vii) $(10x^4y-10xy^4)$

→ Given that, $(10x^4y-10xy^4)$

$(10x^4y-10xy^4) = 10xy\,(x^3-y^3)$

we have, $(a^3-b^3) = (a-b)\,(a^2+ab+b^2)$

$$\boxed{(10x^4y-10xy^4) = 10xy\,(x-y)\,(x^2+xy+y^2)}$$

viii) $54x^6y+2x^3y^4$

$\rightarrow$ Given that, $54x^6y+2x^3y^4$

$54x^6y+2x^3y^4 = 2x^3y\,(27x^3+y^3)$

$= 2x^3y\,[(3x)^3+y^3]$

we have, $(a^3+b^3) = (a+b)(a^2-ab+b^2)$

$$\boxed{54x^6y+2x^3y^4 = 2x^3y\,(3x+y)(9x^2-3xy+y^2)}$$

ix) $32a^3+108b^3$

$\rightarrow$ Here, $32a^3+108b^3 = 4\,(8a^3+27b^3)$

$= 4[(2a)^3+(3b)^3]$

We have, $a^3+b^3 = (a+b)(a^2-ab+b^2)$

$$\boxed{32a^3+108b^3 = 4\,(2a+3b)(4a^2-6ab+9b^2)}$$

x) $(a-2b)^3-512b^3$

$\rightarrow$ Here, $(a-2b)^3-512b^3 = (a-2b)^3-(8b)^3$

we have, $a^3-b^3 = (a-b)(a^2+ab+b^2)$

$\Rightarrow (a-2b)^3-512b^3 = (a-2b-8b)\{(a-2b)^2+(a-2b)8b+(8b)^2\}$

$= (a-10b)\,(a^2+4b^2-4ab+8ab-16b^2+64b^2)$

$$\boxed{(a-2b)^3-512b^3 = (a-10b)(a^2+52b^2+4ab)}$$

xi) $(a+b)^3 - 8(a-b)^3$

→ Here, $(a+b)^3 - 8(a-b)^3$

$= (a+b)^3 - [2(a-b)]^3$

we have, $(a^3-b^3) = (a-b)(a^2+ab+b^2)$

$\Rightarrow$ put $a+b = p$ & $q = 2(a-b)$

$\Rightarrow (a+b)^3 - [2(a-b)]^3 = p^3 - q^3$

$= (p-q)(p^2+pq+q^2)$ —— ①

Now, $p-q = a+b-2(a-b)$

$= a+b-2a+2b$

$p-q = 3b-a$ —— ②

And $p^2+pq+q^2 = (a+b)^2 + (a+b)(2a-2b) + 4(a-b)^2$

$= a^2+2ab+b^2+2a^2-2ab+2ba-2b^2+4(a^2-2ab+b^2)$

$= 3a^2+2ab-b^2+4a^2-8ab+4b^2$

$p^2+pq+q^2 = 7a^2+3b^2-6ab$ —— ③

from equ^n ② & ③

equ^n ① $\Rightarrow$

$(a+b)^3 - [2(a-b)]^3 = (p-q)(p^2+pq+q^2)$

$$\boxed{(a+b)^3 - 8(a-b)^3 = (3b-a)(7a^2+3b^2-6ab)}$$

Q.12 xii) $(x+2)^3+(x-2)^3$

$\rightarrow$ Here, $(x+2)^3+(x-2)^3$

put $x+2=p$ & $x-2=q$

$\Rightarrow (x+2)^3+(x-2)^3 = p^3+q^3$

we have, $(a^3+b^3) = (a+b)(a^2-ab+b^2)$

$\Rightarrow (x+2)^3+(x-2)^3 = p^3+q^3$

$= (p+q)(p^2-pq+q^2)$ —— ①

Now, $p+q = x+2+x-2 = 2x$ —— ②

And $p^2-pq+q^2 = (x+2)^2-(x+2)(x-2)+(x-2)^2$

$= x^2+4x+4-(x^2-4)+x^2-4x+4$

$p^2-pq+q^2 = x^2+12$ —— ③

from equn ①, ② & ③,

$\Rightarrow (x+2)^3+(x-2)^3 = (p+q)(p^2-pq+q^2)$

$$\boxed{(x+2)^3+(x-2)^3 = 2x(x^2+12)}$$

Exercise 5.3

Q.1.) Factorize $64a^3+125b^3+240a^2b+300ab^2$.

→ Given that, $64a^3+125b^3+240a^2b+300ab^2$

which can rewritten as,

$= (4a)^3+(5b)^3+3(4a^2)(5b)+3(4a)(5b)^2$.

$\because (a^3+b^3)+3a^2b+3ab^2=(a+b)^3$

$\Rightarrow \boxed{64a^3+125b^3+240a^2b+300ab^2=(4a+5b)^3}$

Q.2.) Factorize $125x^3-27y^3-225x^2y+135xy^2$.

→ Given that, $125x^3-27y^3-225x^2y+135xy^2$.

which can be rewritten as,

$= (5x)^3-(3y)^3-3(5x)^2(3y)+3(5x)(3y)^2$

But we have, $a^3-b^3-3a^2b+3ab^2=(a-b)^3$

$\Rightarrow \boxed{125x^3-27y^3-225x^2y+135xy^2=(5x-3y)^3}$

Q.3) Factorize $8/27x^3+1+4/3x^2+2x$.

→ Given that, $\frac{8}{27}x^3+1+\frac{4}{3}x^2+2x$

which can be written as,

$= \left(\frac{2}{3}x\right)^3+(1)^3+3\left(\frac{2}{3}x\right)^2(1)+3(1)^2\left(\frac{2}{3}x\right)$

But, $(a+b)^3=a^3+b^3+3a^2b+3ab^2$

$\Rightarrow \boxed{\frac{8}{27}x^3+1+\frac{4}{3}x^2+2x=\left(\frac{2}{3}x+1\right)^3}$

q.4) Factorize $8x^3+27y^3+36x^2y+54xy^2$

Soln:- Given that, $8x^3+27y^3+36x^2y+54xy^2$

which can be written as,

$$= (2x)^3+(3y)^3+3(2x)^2(3y)+3(2x)(3y)^2$$

$$\because (a+b)^3 = a^3+b^3+3a^2b+3ab^2$$

$$\Rightarrow \boxed{(8x^3+27y^3+36x^2y+54xy^2) = (2x+3y)^3}$$

q.5) Factorize $a^3-3a^2b+3ab^2-b^3+8$

Soln:- Given that, $a^3-3a^2b+3ab^2-b^3+8$

$$\Rightarrow (a^3-b^3-3a^2b+3ab^2)+8$$

$\because (a+b)^3 = a^3+b^3+3a^2b+3ab^2$

$$= (a-b)^3+2^3$$

$\because (a-b)^3 = a^3-b^3-3a^2b+3ab^2$

$$= (a-b+2)[(a-b)^2-(a-b)2+4]$$

$\because a^3+b^3 = (a+b)(a^2-ab+b^2)$

$$= (a-b+2)[a^2-2ab+b^2-2a+2b+4]$$

$$= (a-b+2)(a^2+b^2-2ab-2a+2b+4)$$

$$\Rightarrow \boxed{a^3-3a^2b+3ab^2-b^3+8 = (a-b+2)(a^2+b^2-2ab-2a+2b+4)}$$

Exercise 5.4

Q.1) Factorize each of the following expressions.

i) $a^3+8b^3+64c^3-24abc$

$\rightarrow$ Here, $a^3+8b^3+64c^3-24abc$

$= (a)^3+(2b)^3+(4c)^3-3(a)(2b)(4c)$

we have, $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$\Rightarrow (a+2b+4c)[a^2+4b^2+16c^2-a(2b)-(2b)(4c)-4c(a)]$

$= (a+2b+4c)[a^2+4b^2+16c^2-2ab-8bc-4ac]$

$= (a+2b+4c)(a^2+4b^2+16c^2-2ab-8bc-4ac)$

$\Rightarrow \boxed{(a^3+8b^3+64c^3-24abc) = (a+2b+4c)(a^2+4b^2+16c^2-2ab-8bc-4ac)}$

ii) $x^3-8y^3+27z^3+18xyz$

$\rightarrow$ Here, $x^3-8y^3+27z^3+18xyz$

$= (x)^3-(2y)^3+(3z)^3-3(x)(-2y)(3z)$

we have, $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$\Rightarrow [x-2y+3z][x^2+(-2y)^2+(3z)^2-x(-2y)-(-2y)(3z)-3z(x)]$

$= (x-2y+3z)(x^2+4y^2+9z^2+2xy+6yz-3zx)$

Thus, $\boxed{(x^3-8y^3+27z^3+18xyz) = (x-2y+3z)(x^2+4y^2+9z^2+2xy+6yz-3zx)}$

iii) $27x^3-y^3-z^3-9xyz$

$\rightarrow$ Here, $27x^3-y^3-z^3-9xyz$

$= (3x)^3-(y)^3-(z)^3-3(3xyz)$

we have, $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$\Rightarrow (3x-y-z)[(3x)^2+(-y)^2+(-z)^2+3xy-yz+3xz]$

$= (3x-y-z)(9x^2+y^2+z^2+3xy-yz+3xz)$

$\Rightarrow \boxed{(27x^3-y^3-z^3-9xyz) = (3x-y-z)(9x^2+y^2+z^2+3xy-yz+3xz)}$

iv) $\frac{1}{27}x^3-y^3+125z^3+5xyz$.

$\rightarrow$ Here, $\frac{1}{27}x^3-y^3+125z^3+5xyz$

we have, $\left(\frac{x}{3}\right)^3+(-y)^3+(5z)^3-3(x/3)(-y)(5z)$

But, $(a^3+b^3+c^3-3abc) = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$.

$\Rightarrow [x/3-y+5z][(x/3)^2+(-y)^2+(5z)^2-x/3(-y)-(-y)(5z)-5z(x/3)]$

$= (x/3-y+5z)\left(\frac{x^2}{9}+y^2+25z^2+xy/3+5yz-5zx/3\right)$

Thus, $\boxed{\left(\frac{x^3}{27}-y^3+125z^2+5xyz\right) = \left(\frac{x}{3}-y+5z\right)\left(\frac{x^2}{9}+y^2+25z^2+\frac{xy}{3}+5yz-\frac{5zx}{3}\right)}$

v) $8x^3+27y^3-216z^3+108xyz$

→ Here, $8x^3+27y^3-216z^3+108xyz$

$= (2x)^3+(3y)^3+(-6y)^3-3(2x)(3y)(-6z)$

$\Rightarrow \because (a^3+b^3+c^3-3abc) = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$\Rightarrow (2x+3y-6z)\left[(2x)^2+(3y)^2+(-6z)^2-2x(3y)-3y(-6z)-(-6z)2x\right]$

$= (2x+3y-6z)(4x^2+9y^2+36z^2-6xy+18yz+12zx)$

Thus, $\boxed{8x^3+27y^3-216z^3+108xyz = (2x+3y-6z)(4x^2+9y^2+36z^2-6xy+18yz+12zx)}$

vi) $125+8x^3-27y^3+90xy$

→ Here, $125+8x^3-27y^3+90xy$

$= (5)^3+(2x)^3+(-3y)^3-3(5)(2x)(-3y)$

we have, $(a^3+b^3+c^3-3abc) = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$\Rightarrow [5+2x+(-3y)][5^2+(2x)^2+(-3y)^2-5(2x)-2x(-3y)-(-3y)(5)]$

$= (5+2x-3y)(25+4x^2+9y^2-10x+6xy+15y)$

Thus, $\boxed{125+8x^3-27y^3+90xy = (5+2x-3y)(25+4x^2+9y^2-10x+6xy+15y)}$

vii) $(3x-2y)^3+(2y-4z)^3+(4z-3x)^3$

Soln:- Here, $(3x-2y)^3+(2y-4z)^3+(4z-3x)^3$

Let us suppose, $3x-2y=a$, $2y-4z=b$ & $(4z-3x)=c$

$\Rightarrow \quad a^3+b^3+c^3=(3x-2y)^3+(2y-4z)^3+(4z-3x)^3$

But we have, $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca$

Here, $a+b+c=3x-2y+2y-4z+4z-3x$

$\boxed{a+b+c=0}$

$\Rightarrow \quad a^3+b^3+c^3-3abc=0$

$\Rightarrow a^3+b^3+c^3=3abc$

$\Rightarrow a^3+b^3+c^3=3(3x-2y)(2y-4z)(4z-3x)$

Thus, $\boxed{(3x-2y)^3+(2y-4z)^3+(4z-3x)^3=3(3x-2y)(2y-4z)(4z-3x)}$

viii) $(2x-3y)^3+(4z-2x)^3+(3y-4z)^3$

$\rightarrow$ Here, $(2x-3y)^3+(4z-2x)^3+(3y-4z)^3$

Let us suppose, $a=2x-3y$, $b=4z-2x$, $c=3y-4z$

$\Rightarrow a^3+b^3+c^3=(2x-3y)^3+(4z-2x)^3+(3y-4z)^3$

But, $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

Here, $a+b+c=2x-3y+4z-2x+3y-4z$

$\boxed{a+b+c=0}$

$\Rightarrow a^3+b^3+c^3-3abc=0$

$\Rightarrow a^3+b^3+c^3=3abc$

Thus, $\boxed{(2x-3y)^3+(4z-2x)^3+(3y-4z)^3=3(2x-3y)(4z-2x)(3y-4z)}$

Exercise VSAQs

Q.1) Factorize x^4+x^2+25

$\rightarrow$ Here, x^4+x^2+25

$\Rightarrow (x^2)^2+5^2+x^2$

we have, $a^2+b^2=(a+b)^2-2ab$

$$\Rightarrow (x^2)^2+5^2+x^2 = (x^2+5)^2-2(x^2)(5)+x^2$$
$$= (x^2+5)^2-10x^2+x^2$$
$$= (x^2+5)^2-9x^2$$
$$= (x^2+5)^2-(3x)^2$$

Again we have, $(a^2-b^2)=(a-b)(a+b)$

$$\Rightarrow \boxed{(x^2)^2+5^2+x^2 = (x^2+5+3x)(x^2-3x+5)}$$

Q.2) Factorize $x^2-1-2a-a^2$

$\rightarrow$ Here, $x^2-1-2a-a^2$

$= x^2-(1+2a+a^2)$

$= x^2-(a+1)^2 \quad \because (a^2+b^2+2ab)=(a+b)^2$

$= (x+a+1)(x-a-1) \quad \because a^2-b^2=(a-b)(a+b)$

Thus, $\boxed{x^2-1-2a-a^2=(x+a+1)(x-a-1)}$

Q.3) If $a+b+c=0$ then write the value of $a^3+b^3+c^3$.

$\longrightarrow$ we have, $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

But, given that $a+b+c=0$

$\Rightarrow \quad a^3+b^3+c^3-3abc=0$

$\Rightarrow \quad \boxed{a^3+b^3+c^3=3abc}$

Q.4) If $a^2+b^2+c^2=20$ & $a+b+c=0$, find $ab+bc+ca$.

$\longrightarrow$ we have, $(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

Here, given that $a^2+b^2+c^2=20$ & $a+b+c=0$

$\Rightarrow \quad 0=a^2+b^2+c^2+2(ab+bc+ca)$

$\Rightarrow \quad 0=20+2(ab+bc+ca)$

$\Rightarrow \quad 2(ab+bc+ca)=-20$

$\boxed{(ab+bc+ca)=-10}$

Q.5) If $a+b+c=9$ & $ab+bc+ca=40$, find $a^2+b^2+c^2$

$\longrightarrow$ Here, $a+b+c=9$ & $ab+bc+ca=40$

we have, $(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

$9^2=a^2+b^2+c^2+2(40)$

$81=a^2+b^2+c^2+80$

$a^2+b^2+c^2=81-80$

$\boxed{a^2+b^2+c^2=1}$