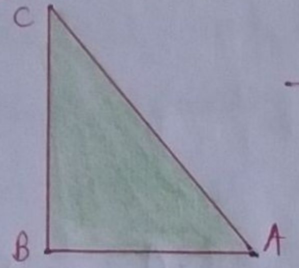
Chapter 5: Trigonometric Ratios

Exercise 5.1

Q.1.) In each of the following, one of the six trigonometric ratios given. Find the values of the other trigonometric ratios.



i) $\sin A = 2/3$

→ Here, given that

$$\sin A = 2/3 \quad \text{—— ①}$$

According to definition,

$$\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{2}{3} \quad \text{—— ②}$$

from ① and ② ⇒ Opposite side = 2

And Hypotenuse = 3

According to Paythagoras theorem,

In ΔABC, $AC^2 = AB^2 + BC^2$

$\Rightarrow 3^2 = AB^2 + 2^2$

$AB^2 = 9 - 4$

$AB^2 = 5$

$\boxed{AB = \sqrt{5}}$ Thus, base $= \sqrt{5}$.

Now, according to definition,

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}}$$

$\cos A = \sqrt{5}/3$

Since, $\operatorname{cosec} A = 1/\sin A$ = Hypotenuse/perpendicular

$\operatorname{cosec} A = 3/2$

And $\sec A$ = Hypotenuse / Base

$\boxed{\sec A = 3/\sqrt{5}}$

tanA = Perpendicular / Base

$\boxed{\tan A = 2/\sqrt{5}}$

Also, cotA = 1/tan A

= Base / perpendicular

$\boxed{\cot A = \sqrt{5}/2}$

ii) cosA = 4/5

→ Here given that,

cos A = 4/5

But, cos A = Base/Hypotenuse

⇒ Base = 4 & Hypotenuse = 5

Now, By Paythagoras theorem,

In $\triangle ABC$, $AC^2 = AB^2 + BC^2$

$\Rightarrow 5^2 = 4^2 + BC^2$

$BC^2 = 25 - 16$

$BC^2 = 9$

$\boxed{BC = 3}$

Thus, perpendicular = 3

Now, sin A = perpendicular / hypotenuse

$\boxed{\sin A = 3/5}$

Now, tanA = perpendicular / Base

tanA = 3/4

Now, cotA = 1/tan A = Base/ perpendicular

$\therefore \boxed{\cot A = 4/3}$

$\operatorname{cosec} A = \frac{1}{\sin A}$

cosecA = 1/(3/5)

$\boxed{\operatorname{cosec} A = 5/3}$

But, cosecA = $\frac{\text{Hypotenuse}}{\text{perpendicular}}$

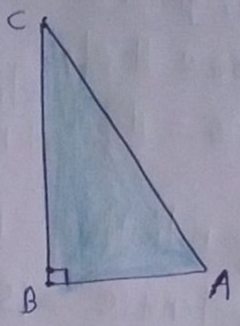
secA = 1/cosA

secA = $\frac{\text{Hypoteneuse}}{\text{Base}}$

$\boxed{\sec A = 5/4}$

iii) $\tan\theta = 11/1$

→ Here, given that $\tan\theta = \frac{11}{1}$ —①



But, we know that

$$\tan\theta = \frac{\text{apposite side}}{\text{adjacent side}} \quad —②$$

⇒ from ① and ②

Base = 1 and perpendicular = 11

Now, According to Paythagoras theorem,

In ΔABC, $AC^2 = AB^2 + BC^2$

$AC^2 = 1^2 + 11^2$

$AC^2 = 1 + 121$

$AC^2 = 122$

$\boxed{AC = \sqrt{122}}$ ⇒ Hypotenuse $= \sqrt{122}$

Now, $\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{11}{\sqrt{122}}$

$\text{cosec}\theta = \frac{1}{\sin\theta}$

$\boxed{\text{cosec}\theta = \frac{\sqrt{122}}{11}}$

Again, $\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}$

$\boxed{\cos\theta = 1/\sqrt{122}}$

And $\sec\theta = 1/\cos\theta$

$\boxed{\sec\theta = \sqrt{122}}$

$\cot\theta = 1/\tan\theta$

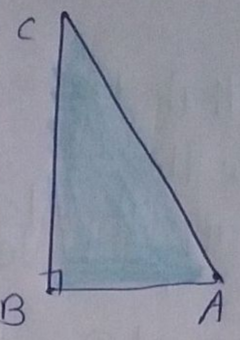
$\boxed{\cot\theta = 1/11}$

iv) $\sin\theta = 11/15$

→ Here, given that $\sin\theta = 11/15$

we know that,

$$\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$$



⇒ perpendicular $= 11$ and

hypotenuse $= 15$

Now, In ΔABC, By Paythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 15^2 = AB^2 + 11^2$$

$$225 = AB^2 + 121$$

$$AB^2 = 225 - 121$$

$$AB^2 = 104$$

$$\boxed{AB = \sqrt{104}} \Rightarrow \text{Base} = \sqrt{104}$$

$$\text{Base} = 2\sqrt{26}$$

$$\cos\theta = \frac{\text{Base}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{2\sqrt{26}}{15}$$

$$\text{cosec}\theta = \frac{1}{\sin\theta}$$

$$\boxed{\text{cosec}\theta = 15/11}$$

$\sec\theta =$ Hypotenuse/Base

$$\boxed{\sec\theta = 15/2\sqrt{26}}$$

$$\tan\theta = \frac{\text{perpendicular}}{\text{Base}}$$

$$\boxed{\tan\theta = 11/2\sqrt{26}}$$

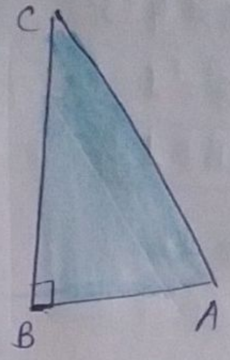
And $\cot\theta = 1/\tan\theta$

$$\boxed{\cot\theta = 2\sqrt{26}/11}$$

v) $\tan\alpha = 5/12$

→ Here, given that, $\tan\alpha = 5/12$

we know that, $\tan\alpha = \frac{\text{perpendicular}}{\text{Base}}$



$\Rightarrow$ perpendicular $= 5$

Base $= 12$

According to Paythagoras theorem,

In ΔABC,

$AC^2 = AB^2 + BC^2$

$AC^2 = 12^2 + 5^2$

$AC^2 = 144 + 25$

$AC^2 = 169$

$\boxed{AC = 13}$

$\Rightarrow$ Hypotenuse $= 13$

$\sin\alpha = \frac{\text{perpendicular}}{\text{hypotenuse}}$

$\boxed{\sin\alpha = 5/13}$

$\operatorname{cosec}\alpha = 1/\sin\alpha$

$\boxed{\operatorname{cosec}\alpha = 13/5}$

$\cot\alpha = 1/\tan\alpha$

$\boxed{\cot\alpha = 12/5}$

$\cos\alpha =$ Base / hypotenuse

$\boxed{\cos\alpha = 12/13}$

Now, $\sec\alpha = 1/\cos\alpha$

$\boxed{\sec\alpha = 13/12}$

vi) $\sin\theta = \sqrt{3}/2$

→ Here, given that, $\sin\theta = \sqrt{3}/2$

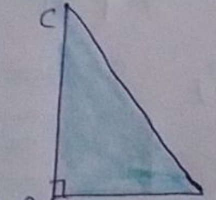
But, we know that, $\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$

$\sin\theta \Rightarrow$ perpendicular $= \sqrt{3}$, hypotenuse $= 2$

By Paythagoras theorem,

In ΔABC, $AC^2 = AB^2 + BC^2$

$2^2 = AB^2 + \sqrt{3}^2$



$4 = AB^2 + 3$

$AB^2 = 4 - 3$

$AB^2 = 1$

$\boxed{AB = 1}$

$\Rightarrow$ Base $= 1$

Now,

$cosec\theta = 1/\sin\theta$

$\boxed{cosec\theta = 2/\sqrt{3}}$

$\cos\theta =$ Base / hypotenuse

$\boxed{\cos\theta = 1/2}$

Now, $\sec\theta = 1/\cos\theta$

$\boxed{\sec\theta = 2}$

$\tan\theta =$ perpendicular / Base

$\boxed{\tan\theta = \sqrt{3}}$

And $\cot\theta = 1/\tan\theta$

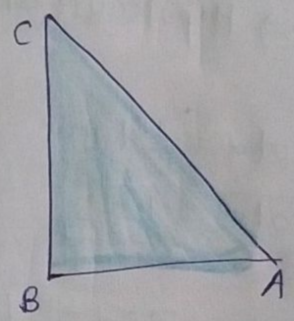
$\boxed{\cot\theta = 1/\sqrt{3}}$

vii) $\cos\theta = 7/25$

$\rightarrow$ Here, given that $\cos\theta = 7/25$

we know that, $\cos\theta = \frac{\text{Base}}{\text{hypotenuse}}$

$\Rightarrow$ Base $= 7$ and hypotenuse $= 25$



According to Paythagoras theorem,

In ABC, $AC^2 = AB^2 + BC^2$

$25^2 = 7^2 + BC^2$

$625 = 49 + BC^2$

$BC^2 = 625 - 49$

$BC^2 = 576$

$\boxed{BC = 24}$

$\Rightarrow$ perpendicular $= 24$

$\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$

$\boxed{\sin\theta = 24/25}$

$cosec\theta = 1/\sin\theta$

$\boxed{cosec\theta = 25/24}$

Now, $\sec\theta = 1/\cos\theta$

$\boxed{\sec\theta = 25/7}$

$\tan\theta = \frac{\text{perpendicular}}{\text{Base}}$

$\boxed{\tan\theta = 24/7}$

$\cot\theta = 1/\tan\theta$

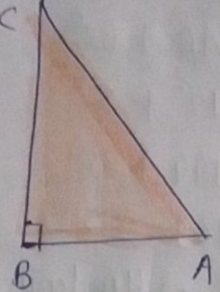
$\boxed{\cot\theta = 7/24}$

viii) $\tan\theta = 8/15$

→ Here, given that $\tan\theta = 8/15$

But, $\tan\theta = \frac{\text{perpendicular}}{\text{Base}}$

perpendicular = 8 , Base = 15



According to Paythagoras theorem,

In ABC, $AC^2 = AB^2 + BC^2$

$AC^2 = 15^2 + 8^2$

$AC^2 = 225 + 64$

$AC^2 = 289$

$AC = \sqrt{289}$

$\boxed{AC = 17}$

⇒ Hypotenuse = 17

$\cot\theta = 1/\tan\theta$

$\boxed{\cot\theta = 15/8}$

$\sin\theta = \frac{\text{perpendicular}}{\text{Hypotenuse}}$

$\boxed{\sin\theta = 8/17}$

$\text{cosec}\,\theta = 1/\sin\theta$

$\boxed{\text{cosec}\,\theta = 17/8}$

$\cos\theta = \text{Base}/\text{hypotenuse}$

$\boxed{\cos\theta = 15/17}$

$\sec\theta = 1/\cos\theta$

$\boxed{\sec\theta = 17/15}$

ix) $\cot\theta = 12/5$

→ Here $\cot\theta = 12/5$

But, $\cot\theta$ = Base/perpendicular

⇒ Base = 12, perpendicular = 5

By Paythagoras theorem,

$AC^2 = AB^2 + BC^2$

$AC^2 = 12^2 + 5^2$

$AC^2 = 144 + 25$

$AC^2 = 169$

$\boxed{AC = 13}$ ⇒ Hypotenuse = 13

Now, $\tan\theta = 1/\cot\theta$

$\boxed{\tan\theta = 5/12}$

$\sin\theta$ = perpendicular / hypotenuse

$\boxed{\sin\theta = 5/13}$

$\operatorname{cosec}\theta = 1/\sin\theta$

$\boxed{\operatorname{cosec}\theta = 13/5}$

$\cos\theta$ = Base / hypotenuse

$\boxed{\cos\theta = 12/13}$

$\sec\theta = 1/\cos\theta$

$\boxed{\sec\theta = 13/12}$

x) $\sec\theta = 13/5$

→ Here $\sec\theta = 13/5$

But, $\sec\theta = \dfrac{\text{Hypotenuse}}{\text{Base}}$

⇒ Hypotenuse = 13,

Base = 5

According to Paythagoras thm,

$AC^2 = AB^2 + BC^2$

$13^2 = 5^2 + BC^2$

$169 = 25 + BC^2$

$169 - 25 = BC^2$

$144 = BC^2$ ⇒ $\boxed{BC = 12}$

perpendicular = 12

Now, $\cos\theta = 1/\sec\theta$

$\boxed{\cos\theta = 5/13}$

$\sin\theta = \dfrac{\text{perpendicular}}{\text{hypotenuse}}$

$\boxed{\sin\theta = 12/13}$

$\operatorname{cosec}\theta = 1/\sin\theta$

$\boxed{\operatorname{cosec}\theta = 13/12}$

$\tan\theta$ = perpendicular / Base

$\boxed{\tan\theta = 12/5}$

$\cot\theta = 1/\tan\theta$

$\boxed{\cot\theta = 5/12}$

xi) $\csc\theta = \sqrt{10}$

→ Here, $\csc\theta = \sqrt{10}$

But, $\csc\theta = \dfrac{\text{Hypotenuse}}{\text{perpendicular}}$

∴ Hypotenuse $= \sqrt{10}$
perpendicular $= 1$

By Paythagoras theorem,
In ΔABC,

$AC^2 = AB^2 + BC^2$

$\sqrt{10}^2 = AB^2 + 1^2$

$10 = AB^2 + 1$

$AB^2 = 10 - 1$

$AB^2 = 9 \Rightarrow \boxed{AB = 3}$

$\Rightarrow$ Base $= 3$

Now, $\sin\theta = 1/\csc\theta$

$\boxed{\sin\theta = 1/\sqrt{10}}$

$\cos\theta = \text{Base}/\text{hypotenuse}$

$\boxed{\cos\theta = 3/\sqrt{10}}$

$\sec\theta = 1/\cos\theta$

$\boxed{\sec\theta = \sqrt{10}/3}$

$\tan\theta = \dfrac{\text{perpendicular}}{\text{Base}}$

$\boxed{\tan\theta = 1/3}$

$\boxed{\cot\theta = 3}$

xii) $\cos\theta = 12/15$

→ Here, $\cos\theta = 12/15$

But, $\cos\theta = \dfrac{\text{Base}}{\text{hypotenuse}}$

$\Rightarrow$ Base $= 12$ and
hypotenuse $= 15$

By Paythagoras theorem,
In ABC,

$AC^2 = AB^2 + BC^2$

$15^2 = 12^2 + BC^2$

$225 - 144 = BC^2$

$BC^2 = \sqrt{81}^2$

$\boxed{BC = 9}$

$\Rightarrow$ perpendicular $= 9$

Now, $\sec\theta = 1/\cos\theta$

$\boxed{\sec\theta = 15/12}$

$\sin\theta = \dfrac{\text{perpendicular}}{\text{hypotenuse}}$

$\boxed{\sin\theta = 9/15}$

$\csc\theta = 1/\sin\theta$

$\boxed{\csc\theta = 15/9}$

$\tan\theta = \text{perpendicular}/\text{Base}$

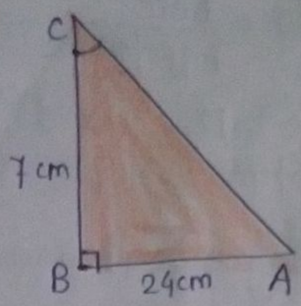
$\boxed{\tan\theta = 9/12}$

$\cot\theta = 1/\tan\theta$

$\boxed{\cot\theta = 12/9}$

Q.2. In a ΔABC, right angled at B, AB = 24 cm, BC = 7cm. determine i) sinA, cosA ii) sinC, cosC.

→



i) In ΔABC,

$\angle CBA = 90°$, AB = 24cm, BC = 7cm

According to Paythagoras theorem,

$AC^2 = AB^2 + BC^2$

$AC^2 = 24^2 + 7^2$

$AC^2 = 576 + 49$

$AC^2 = 625$

$\boxed{AC = 25}$

⇒ Hypotenuse = 5

Now, $\sin A = \frac{\text{opposite side}}{\text{hypotenuse}}$

$\sin A = BC/AC$

$\boxed{\sin A = 7/25}$

$\cos A = \frac{\text{adjacent side}}{\text{hypotenuse}}$

$\cos A = AB/AC$

$\boxed{\cos A = 24/25}$

$\operatorname{cosec} A = 1/\sin A$

$\boxed{\operatorname{cosec} A = 25/7}$

$\sec A = 1/\cos A$

$\boxed{\sec A = 25/24}$

Now, $\sin A \cdot \cos A = \frac{7}{25} \times \frac{24}{25}$

ii) In ΔABC,

$\angle CBA = 90°$, AB = 24cm, BC = 7cm, AC = 25cm

$\sin C$ = opposite side/hypotenuse

$\sin C = AB/AC$

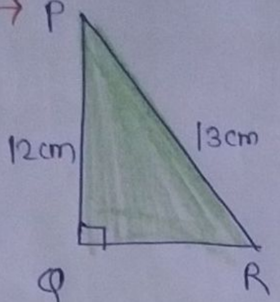
$\boxed{\sin C = 24/25}$

$\cos C = \frac{\text{~~Base~~ adjacent side}}{\text{hypotenuse}}$

$\cos C = BC/AC$

$\boxed{\cos C = 7/25}$

Q.3. In fig. 5.37, find tanP and cotR. Is tanP = cotR?

→ 

Here, given that PR = 13cm, PQ = 12cm

In PQR, ∠PQR = 90°

By Paythagoras theorem,

$PR^2 = QR^2 + PQ^2$

$PR^2 = QR^2 + 12^2$

$169 - 144 = QR^2$

$25 = QR^2 \Rightarrow \boxed{QR = 5\text{ cm}}$

Now,

$\tan P = \dfrac{\text{opposite side to angle P}}{\text{adjdcent side to angle P}}$

$\tan P = QR/PQ$

$\boxed{\tan P = 5/12}$

Now, $\cot R = \dfrac{\text{adjacent side to angle R}}{\text{opposite side to angle R}}$

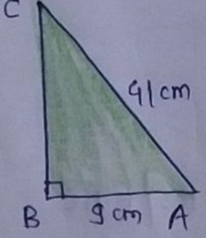
$\cot R = QR/PQ$

$\boxed{\cot R = 5/12}$

Thus, here $\boxed{\tan P = \cot R}$

Hence proved.

Q.4. If sinA = 9/41, compute cosA and tanA.

→ 

Here, given that sinA = 9/41

AC = 41 cm, BA = 9 cm

Now, In ΔABC

By Paythagoras theorem,

$AC^2 = AB^2 + BC^2$

$41^2 = 9^2 + BC^2$

$1681 - 81 = BC^2$

$BC^2 = 1600 \Rightarrow \boxed{BC = 40}$

$\boxed{\text{Base} = 40}$

Now, $\cos A = \text{Base}/\text{hypotenuse}$

$\cos A = AB/AC$

$\boxed{\cos A = 40/41}$

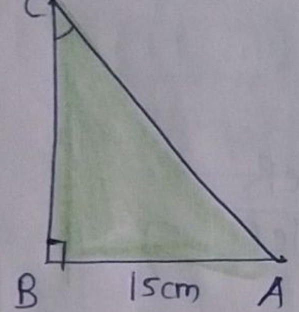
$\tan A = \sin A / \cos A$

$\tan A = \frac{9}{41} \times \frac{41}{40}$

$\boxed{\tan A = 9/40}$

Q.5. Given that: $15 \cot A = 8$, find $\sin A$ and $\sec A$.

Soln:-



Here, given that

$15 \cot A = 8$

$\Rightarrow \boxed{\cot A = 8/15}$

But, $\cot A = \frac{\text{Base}}{\text{perpendicular}}$

Base = 8 cm, perpendicular = 15

$\therefore$ BC = 8 cm, perpendicular = AB = 15 cm

In ΔABC, According to Paythagoras theorem,

$AC^2 = AB^2 + BC^2$

$AC^2 = 15^2 + 8^2$

$AC^2 = 225 + 64$

$AC^2 = 289$

$\boxed{AC = 17}$

$\Rightarrow$ Hypotenuse = 17

Here,

$\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{BC}{AC}$

$\boxed{\sin A = 15/17}$

And $\sec A = 1/\cos A$

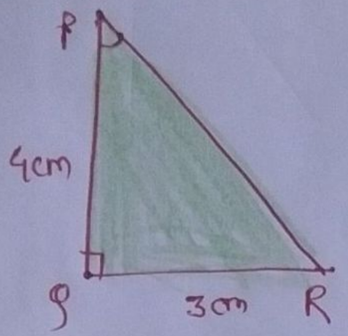
$\sec A = \text{hypotenuse}/\text{Base}$

$\sec A = AC/BC$

$\boxed{\sec A = 17/8}$

Q.6. In ΔPQR, right-angled at Q, PQ = 4cm, RQ = 3cm. Find the value of sinP, sinR, secP and secR.

Solution:-



Here given that

$PQ = 4cm$, $RQ = 3cm$

In ΔPQR, By Paythagoras theorem

$$PR^2 = PQ^2 + QR^2$$

$$PR^2 = 4^2 + 3^2 = 16 + 9 = 25$$

$$\boxed{PR = hypotenuse = 5}$$

Now,

$$\sin P = \frac{\text{opposite side to angle P}}{\text{hypotenuse}} = \frac{QR}{PR} = \frac{3}{5}$$

$$\sec P = 1/\cos P = \frac{\text{hypotenuse}}{\text{adjacent side to angle P}} = \frac{5}{4}$$

Thus, $\boxed{\sin P = 3/5 \ \& \ \sec P = 5/4}$

And,

$$\sin R = \frac{\text{opposite side to angle R}}{\text{hypotenuse}} = \frac{PQ}{PR} = \frac{4}{5}$$

$$\sec R = 1/\cos R = \frac{\text{hypotenuse}}{\text{adjacent side to angle R}} = \frac{5}{3}$$

Thus, $\boxed{\sin R = 4/5 \ \& \ \sec R = 5/3}$

Q.7. If $\cot\theta = 7/8$, evaluate

i) $(1+\sin\theta)(1-\sin\theta) / (1+\cos\theta)(1-\cos\theta)$

ii) $\cot^2\theta$

Solution:-

i) Here, given that $\cot\theta = 7/8$

$$(1+\sin\theta)(1-\sin\theta) = (a+b)(a-b)$$

$$(1-\sin^2\theta) = (a^2-b^2)$$

But, $\sin^2\theta + \cos^2\theta = 1$

$$\Rightarrow 1-\sin^2\theta = \cos^2\theta$$

$$\Rightarrow \boxed{(1+\sin\theta)(1-\sin\theta) = \cos^2\theta}$$

And $(1+\cos\theta)(1-\cos\theta) = (1-\cos^2\theta)$

But, $1-\cos^2\theta = \sin^2\theta$

$$\Rightarrow \boxed{(1+\cos\theta)(1-\cos\theta) = \sin^2\theta}$$

Now, $$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{\cos^2\theta}{\sin^2\theta} = \tan^2\theta$$

$$= (\cot\theta)^2$$

$$= \left(\frac{7}{8}\right)^2$$

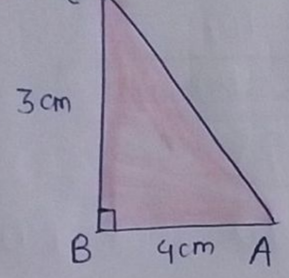
$$\boxed{\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{49}{64}}$$

ii) Also, given that, $\cot\theta = 7/8$

$$\Rightarrow \cot^2\theta = (\cot\theta)^2$$
$$= (7/8)^2$$
$$\boxed{\cot^2\theta = 49/64}$$

Q. 8. If $3\cot A = 4$, check whether $(1-\tan^2 A)/(1+\tan^2 A) = (\cos^2 A - \sin^2 A)$ or not.

Solution :—



Here, given that

$$3\cot A = 4$$
$$\boxed{\cot A = 4/3}$$

$$\tan A = 1/\cot A$$
$$\boxed{\tan A = 3/4}$$

Now, opposite side $= BC = 3cm$

Adjacent side $= AB = 4cm$

In ΔABC, $\angle CBA = 90°$,

By Paythagoras theorem,

$$AB^2 + BC^2 = AC^2$$
$$\Rightarrow AC^2 = 4^2 + 3^2 = 16 + 9 = 25$$
$\boxed{AC = 5}$ hypotenuse $= 5cm$

Now, $\sin A = \dfrac{\text{opposite side}}{\text{hypotenuse}} = \dfrac{ABC}{AC} = \dfrac{3}{5}$

$\cos A = \dfrac{\text{adjacent side}}{\text{hypotenuse}} = \dfrac{AB}{AC} = \dfrac{4}{5}$

Now, $\dfrac{(1-\tan^2 A)}{(1+\tan^2 A)} = \dfrac{1-(3/4)^2}{1+(3/4)^2}$

$$\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-9/16}{1+9/16}$$

$$= \frac{16-9/16}{16+9/16}$$

$$\boxed{\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{7}{25}} \quad \text{—— (1)}$$

Now,
$$\cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

$$= \frac{16}{25} - \frac{9}{25}$$

$$\boxed{\cos^2 A - \sin^2 A = \frac{7}{25}} \quad \text{—— (2)}$$

from (1) and (2),

$$\boxed{\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A}$$

Hence proved.

Q.9. If $\tan\theta = a/b$, find the value of $\frac{(\cos\theta+\sin\theta)}{(\cos\theta-\sin\theta)}$.

Solution:- Here, given that $\tan\theta = a/b$

But, we know that, $\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$

$\Rightarrow$ opposite side $= a$, adjacent side $= b$

By Paythagoras theorem,

$$(\text{opposite side})^2 + (\text{adjacent side})^2 = (\text{hypotenuse})^2$$

$$a^2 + b^2 = (\text{hypotenuse})^2$$

$$\text{hypotenuse} = \sqrt{a^2+b^2}$$

Now, $\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$

$$\boxed{\sin\theta = a/\sqrt{a^2+b^2}}$$

And, $\cos\theta = \text{adjacent side}/\text{hypotenuse}$

$$\boxed{\cos\theta = b/\sqrt{a^2+b^2}}$$

Now, $\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \frac{(a+b)/\sqrt{a^2+b^2}}{(a-b)/\sqrt{a^2+b^2}}$

$$\boxed{\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \frac{a+b}{(a-b)}}$$

Q.10. If $3\tan\theta = 4$, find the value of $\frac{4\cos\theta - \sin\theta}{2\cos\theta + \sin\theta}$.

Solution:- Here, given that

$$3\tan\theta = 4$$

$$\boxed{\tan\theta = 4/3} \Rightarrow \boxed{\cot\theta = 3/4}$$

But, $\frac{4\cos\theta - \sin\theta}{2\cos\theta + \sin\theta} = \frac{(4\cos\theta - \sin\theta)/\cos\theta}{(2\cos\theta + \sin\theta)/\cos\theta}$

$$= \frac{4 - \tan\theta}{2 + \tan\theta}$$

$$= \frac{4 - 4/3}{2 + 4/3} = \frac{(12-4)/3}{(6+4)/3}$$

$$= \frac{8}{10}$$

$$\boxed{\frac{4\cos\theta - \sin\theta}{2\cos\theta + \sin\theta} = 4/5}$$

Q.11. If $3\cot\theta = 2$, find the value of $\frac{4\sin\theta - 3\cos\theta}{2\sin\theta + 6\cos\theta}$.

Solution:– Here, given that $3\cot\theta = 2$

$$\boxed{\cot\theta = 2/3}$$

But, $\cot\theta = 1/\tan\theta$

$$\boxed{\tan\theta = 3/2}$$

Now, $$\frac{4\sin\theta - 3\cos\theta}{2\sin\theta + 6\cos\theta} = \frac{(4\sin\theta - 3\cos\theta)/\sin\theta}{(2\sin\theta + 6\cos\theta)/\sin\theta}$$

$$= \frac{(4 - 3\cot\theta)}{(2 + 6\cot\theta)}$$

$$= \frac{(4 - 3\times 2/3)}{(2 + 6\times 2/3)} = \frac{(4-2)}{(2+4)}$$

$$= \frac{2}{6}$$

$$\boxed{\frac{(4\sin\theta - 3\cos\theta)}{(2\sin\theta + 6\cos\theta)} = 1/3}$$

Q.12. If $\tan\theta = a/b$, prove that $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a^2 - b^2}{a^2 + b^2}$.

Solution:– Here, given that $\boxed{\tan\theta = a/b}$

Now, $$\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{(a\sin\theta - b\cos\theta)/\cos\theta}{(a\sin\theta + b\cos\theta)/\cos\theta}$$

$$= \frac{(a\tan\theta - b)}{(a\tan\theta + b)}$$

$$= \frac{(a\times a/b - b)}{(a\times a/b + b)}$$

$$= \frac{(a^2/b - b)}{(a^2/b + b)}$$

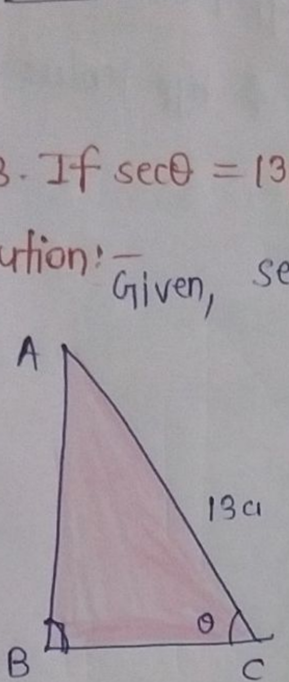
$$= \frac{(a^2-b^2)/b}{(a^2+b^2)/b}$$

$$\boxed{\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a^2-b^2}{a^2+b^2}}$$

Hence proved.

Q.13. If $\sec\theta = 13/5$, show that $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = 3$.

Solution:– Given, $\sec\theta = 13/5$



But, $\sec\theta = 1/\cos\theta$

$$\boxed{\cos\theta = 5/13}$$

But, $\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$

$\cos\theta \Rightarrow$ adjacent side $= BC = 5$

hypotenuse $= AC = 13$

By Paythagoras theorem,

$(\text{adjacent side})^2 + (\text{opposite side})^2 = (\text{hypotenuse})^2$

$$BC^2 + AB^2 = AC^2$$

$$5^2 + AB^2 = 13^2$$

$$25 + AB^2 = 169$$

$$AB^2 = 169 - 25 = 144$$

$\boxed{AB = 12} \Rightarrow$ opposite side $= 12$

Now, $\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{12}{13}$

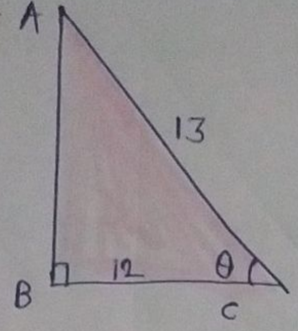
$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{5}{13}$

Now, $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = \frac{2(12/13) - 3(5/13)}{(4)(12/13) - 9(5/13)} = \frac{24-15}{48-45} = \frac{9}{3}$

$= 3$ Hence proved.

14. If $\cos\theta = 12/13$, show that $\sin\theta(1-\tan\theta) = 35/156$.

Soln:-



Given that, $\cos\theta = 12/13$

But, we know that

$$\cos\theta = \frac{BC}{AC} = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$\Rightarrow$ adjacent side $BC = 12$ & hypotenuse $= 13$

But, from ΔABC, In $\angle ABC = 90°$

By Paythagoras theorem,

$$AC^2 = AB^2 + BC^2$$
$$13^2 = AB^2 + 12^2$$
$$AB^2 = 13^2 - 12^2$$
$$AB^2 = 169 - 144 = 25$$
$$AB = \sqrt{25}$$
$$\boxed{AB = 5}$$

Now, $\sin\theta = \frac{\text{opposite side to }\theta}{\text{hypotenuse}}$

$$\sin\theta = \frac{AB}{AC} = \frac{5}{13} \qquad \Rightarrow \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{5/13}{12/13} = \frac{5}{12}$$

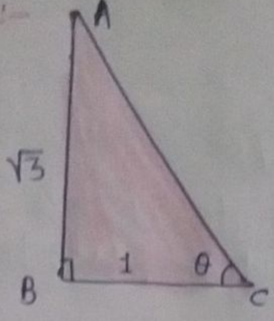
$$\begin{aligned} \text{LHS} &= \sin\theta(1-\tan\theta) \\ &= 5/13\,(1 - 5/12) \\ &= 5/13\,(7/12) \\ &= \frac{35}{156} \end{aligned}$$

LHS = RHS $\qquad \boxed{\sin\theta(1-\tan\theta) = 35/156}$

Hence proved.

15. If $\cot\theta = \frac{1}{\sqrt{3}}$, show that $\frac{1-\cos^2\theta}{2-\sin^2\theta} = \frac{3}{5}$.

Solution:-



Given that, $\cot\theta = \frac{1}{\sqrt{3}}$

But, according to definition,

$$\cot\theta = 1/\tan\theta$$

$$\boxed{\tan\theta = \sqrt{3}}$$

But, $\tan\theta = \frac{\text{opposite side to } \theta}{\text{adjacent side to } \theta} = \frac{\sqrt{3}}{1}$

opposite side $= AB = \sqrt{3}$, adjacent side $= BC = 1$

By Paythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = (\sqrt{3})^2 + 1^2$$

$$AC^2 = 3 + 1 = 4$$

$$\boxed{AC = 2}$$

Now, $\sin\theta = \frac{\text{opposite side to } \theta}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$

$\cos\theta = \frac{\text{adjacent side to } \theta}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{1}{\sqrt{2}}$

Now, L.H.S. $= \frac{1-\cos^2\theta}{1-\sin^2\theta}$

$$= \frac{1-(1/2)^2}{2-3/4} = \frac{1/\cancel{4}}{5/4} = \frac{1}{2}\times\frac{4}{5}$$

$$= \frac{1-1/4}{2-3/4} = \frac{3/4}{5/4}$$

$$= \frac{3}{5}$$

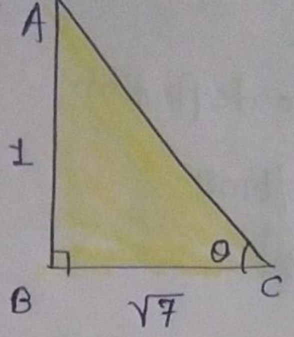
$= R.H.S.$

Thus, $\frac{1-\cos^2\theta}{2-\sin^2\theta} = \frac{3}{5}$

Hence proved.

16. If $\tan\theta = \frac{1}{\sqrt{7}}$, then show that $\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta} = \frac{3}{4}$.

Solution:-



Here, given that

$$\tan\theta = \frac{1}{\sqrt{7}}$$

But, $\tan\theta = \frac{\text{opposite side to } \theta}{\text{addjacent side to } \theta}$

$\Rightarrow$ opposite side $= AB = 1$, adjacent side $= BC = \sqrt{7}$

In ΔABC, $\angle ABC = 90^\circ$

By Paythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 1 + (\sqrt{7})^2$$

$$AC^2 = 1 + 7 = 8$$

$$\Rightarrow \boxed{AC = 2\sqrt{2}}$$

Now, $\sin\theta = \frac{\text{opposite side to } \theta}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{1}{2\sqrt{2}}$

$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{1/2\sqrt{2}} = 2\sqrt{2}$

$\cos\theta = \frac{\text{adjacent side to } \theta}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{\sqrt{7}}{\cancel{1}/2\sqrt{2}}$

$\cancel{\cos\theta = 2\sqrt{14}} \quad \cancel{\cos\theta =}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\sqrt{7}/2\sqrt{2}} = \frac{2\sqrt{2}}{\sqrt{7}}$

Now, LHS $= \frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}$

$$= \frac{(2\sqrt{2})^2 - \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}{(2\sqrt{2})^2 + \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}$$

$$= \frac{8 - 8/7}{8 + 8/7}$$

$$= \frac{48/7}{64/7} = \frac{48}{64} = \frac{3}{4} = \text{R.H.S.}$$

Hence proved.

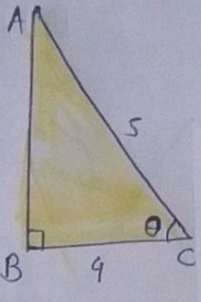
17. If $\sec\theta = 5/4$, find the value of $\dfrac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta}$.

Solution: Given that,

$$\sec\theta = 5/4$$

But, $\sec\theta = 1/\cos\theta \Rightarrow \cos\theta = 1/\sec\theta$

$$\boxed{\cos\theta = 4/5}$$



By definition, $\cos\theta = \dfrac{\text{adjacent side to } \theta}{\text{hypotenuse}}$

adjacent side $= BC = 4$, hypotenuse $= AC = 5$

In ΔABC, $\angle ABC = 90^\circ$

By Paythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = AB^2 + 4^2$$

$$AB^2 = 5^2 + 4^2 = 25 - 16 = 9$$

$$\boxed{AB = 3}$$

Now, $\sin\theta = \dfrac{\text{opposite side}}{\text{hypotenuse}} = \dfrac{AB}{AC} = \dfrac{3}{5}$

$\cos\theta = 4/5$ $\qquad \tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{3/5}{4/5} = \dfrac{3}{4}$

And $\cot\theta = 1/\tan\theta = 1/3/4 = 4/3$

Now, $LHS = \dfrac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta}$

$$= \frac{3/5 - 2(4/5)}{3/4 - 4/3} = \frac{(3-8)/5}{(9-16)/12}$$

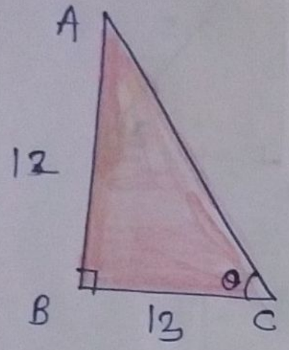
$$= \frac{-5/5}{-7/12} = \frac{1}{7/12}$$

$$\boxed{\frac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta} = \frac{12}{7}}$$

18. If $\tan\theta = 12/13$, find the value of $\frac{2\sin\theta \cdot \cos\theta}{\cos^2\theta - \sin^2\theta}$.

Solution:- Given that, $\tan\theta = 12/13$

But, $\tan\theta = \frac{\text{opposite side to } \theta}{\text{adjacent side to } \theta} = \frac{AB}{BC} = \frac{12}{13}$



opposite side $= AB = 12$

adjacent side $= BC = 13$

$\sin\theta = \frac{\text{opposite side to } \theta}{\text{hypotenuse}} = \frac{AB}{AC}$

In ΔABC, $\angle ABC = 90^\circ$

By Paythagoras theorem,

$AC^2 = AB^2 + BC^2$

$AC^2 = (12)^2 + (13)^2$

$= 144 + 169$

$AC^2 = 313$

$\boxed{AC = \sqrt{313}}$ $\Rightarrow$ $\boxed{\sin\theta = \frac{12}{\sqrt{313}}}$

and $\cos\theta = \frac{\text{adjacent side to } \theta}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{13}{\sqrt{313}}$

Now,

$$\frac{2\sin\theta \cdot \cos\theta}{\cos^2\theta - \sin^2\theta} = \frac{2 \times 12/\sqrt{313} \times 13/\sqrt{313}}{\frac{169}{313} - \frac{144}{313}}$$

$$= \frac{2 \times 12 \times 13}{169 - 144}$$

$$\boxed{\frac{2\sin\theta \cdot \cos\theta}{\cos^2\theta - \sin^2\theta} = \frac{312}{25}}$$

Exercise 5.2

Evaluate each of the following:

1. $\sin 45° \cdot \sin 30° + \cos 45° \cdot \cos 30°$

→

$\sin 45° \cdot \sin 30° + \cos 45° \cdot \cos 30°$

$\because \sin 45° = \frac{1}{\sqrt{2}}$, $\sin 30° = \frac{1}{2}$

$\cos 45° = \frac{1}{\sqrt{2}}$, $\cos 30° = \frac{\sqrt{3}}{2}$

$\sin 45° \cdot \sin 30° + \cos 45° \cdot \cos 30° = \frac{1}{\sqrt{2}} \times \frac{1}{2} + \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}$

$= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$

$\sin 45° \cdot \sin 30° + \cos 45° \cdot \cos 30° = \frac{(1+\sqrt{3})}{2\sqrt{2}}$

2. $\sin 60° \cdot \cos 30° + \cos 60° \cdot \sin 30°$

Solution:- $\sin 60° = \frac{\sqrt{3}}{2}$, $\sin 30° = \frac{1}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$, $\cos 60° = \frac{1}{2}$

Now, $\sin 60° \cdot \cos 30° + \cos 60° \cdot \sin 30°$

$= \frac{\sqrt{3}}{2}\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{3}{4} + \frac{1}{4} = \frac{4}{4}$

$= 1$

$\boxed{\sin 60° \cdot \cos 30° + \cos 60° \cdot \sin 30° = 1}$

3. $\cos 60^\circ \cdot \cos 45^\circ - \sin 60^\circ \sin 45^\circ$

Solution:- $\cos 60^\circ = \frac{1}{2}$, $\cos 45^\circ = \frac{1}{\sqrt{2}}$

$\sin 60^\circ = \frac{\sqrt{3}}{2}$, $\sin 45^\circ = \frac{1}{\sqrt{2}}$

$\cos 60^\circ \cdot \cos 45^\circ - \sin 60^\circ \cdot \sin 45^\circ = \frac{1}{2} \cdot \left(\frac{1}{\sqrt{2}}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{1-\sqrt{3}}{2\sqrt{2}}$

$$\boxed{\cos 60^\circ \cdot \cos 45^\circ - \sin 60^\circ \cdot \sin 45^\circ = \left(\frac{1-\sqrt{3}}{2\sqrt{2}}\right)}$$

4. $\sin^2 30^\circ + \sin^2 45^\circ + \sin^2 60^\circ + \sin^2 90^\circ$

$\longrightarrow$ $\sin 30^\circ = \frac{1}{2}$, $\sin 45^\circ = \frac{1}{\sqrt{2}}$, $\sin 60^\circ = \frac{\sqrt{3}}{2}$, $\sin 90^\circ = 1$

Now, LHS $= \sin^2 30^\circ + \sin^2 45^\circ + \sin^2 60^\circ + \sin^2 90^\circ$

$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + (1)^2$

$= \frac{1}{4} + \frac{1}{2} + \frac{3}{4} + 1$

$= 2 + \frac{1}{2}$

$= \frac{5}{2}$ $\quad \boxed{\therefore \sin^2 30^\circ + \sin^2 45^\circ + \sin^2 60^\circ + \sin^2 90^\circ = 5/2}$

5. $\cos^2 30^\circ + \cos^2 45^\circ + \cos^2 60^\circ + \cos^2 90^\circ$

solution:- $\cos^2 30^\circ + \cos^2 45^\circ + \cos^2 60^\circ + \cos^2 90^\circ$

we know that, $\cos 30^\circ = \frac{\sqrt{3}}{2}$, $\cos 45^\circ = \frac{1}{\sqrt{2}}$, $\cos 60^\circ = \frac{1}{2}$

$\cos 90^\circ = 0$

Then, $\cos^2 30^\circ + \cos^2 45^\circ + \cos^2 60^\circ + \cos^2 90^\circ$

$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + 0$

$= \frac{3}{4} + \frac{1}{2} + \frac{1}{4}$

$= 3/2$

6. $\tan^2 30° + \tan^2 45° + \tan^2 60°$

Solution:- we know that,

$\tan 30° = \frac{1}{\sqrt{3}}$, $\tan 60° = \sqrt{3}$, $\tan 45° = 1$

Now, $\tan^2 30° + \tan^2 45° + \tan^2 60° = \left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{3})^2 + 1^2$

$= \frac{1}{3} + 3 + 1$

$= \frac{1}{3} + 4$

$$\boxed{\tan^2 30° + \tan^2 45° + \tan^2 60° = \frac{13}{3}}$$

7. $2\sin^2 30° - 3\cos^2 45° + \tan^2 60°$

solution:- $\sin 30° = \frac{1}{2}$, $\cos 45° = \frac{1}{\sqrt{2}}$, $\tan 60° = \sqrt{3}$

Now, $2\sin^2 30° - 3\cos^2 45° + \tan^2 60° = 2\left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{\sqrt{2}}\right)^2 + (\sqrt{3})^2$

$= 2\left(\frac{1}{4}\right) - 3\left(\frac{1}{2}\right) + 3$

$= \frac{1}{2} - \frac{3}{2} + 3$

$= -1 + 3$

$$\boxed{2\sin^2 30° - 3\cos^2 45° + \tan^2 60° = 2}$$

8. $\sin^2 30° \cdot \cos^2 45° + 4\tan^2 30° + (1/2)\sin^2 90° - 2\cos^2 90° + (1/24)\cos 2 0°$

solution:- we know that,

$\sin 30° = 1/2$, $\cos 45° = 1/\sqrt{2}$, $\tan 30° = \frac{1}{\sqrt{3}}$, $\sin 90° = 1$

$\cos 90° = 0$, $\cos^2 0° = 1$

Now,

$\sin^2 30° \cdot \cos^2 45° + 4\tan^2 30° + (1/2)\sin^2 90° - 2\cos^2 90° + (1/24)\cos^2 0°$

$= (1/2)^2 (1/\sqrt{2})^2 + 4\left(\frac{1}{\sqrt{3}}\right)^2 + \frac{1}{2}(1)^2 - 2(0) + \frac{1}{24}(1)^2$

$= \frac{1}{8} + \frac{4}{3} + \frac{1}{2} + \frac{1}{24} = \frac{48}{24} = 2$

9. $4(\sin^4 60° + \cos^4 30°) - 3(\tan^2 60° - \tan^2 45°) + 5\cos^2 45°$

solution:- We know that,

$\sin 60° = \frac{\sqrt{3}}{2}$, $\cos 30° = \frac{\sqrt{3}}{2}$

$\tan 60° = \sqrt{3}$, $\cos 45° = \frac{1}{\sqrt{2}}$, $\tan 45° = 1$

Now, $4(\sin^4 60° + \cos^4 30°) - 3(\tan^2 60° - \tan^2 45°) + 5\cos^2 45°$

$= 4\left[\left(\frac{\sqrt{3}}{2}\right)^4 + \left(\frac{\sqrt{3}}{2}\right)^4\right] - 3\left[(\sqrt{3})^2 - (1)^2\right] + 5\left(\frac{1}{\sqrt{2}}\right)^2$

$= 4\left[\frac{9}{16} + \frac{9}{16}\right] - 3[3-1] + \frac{5}{2}$

$= 4\left(\frac{18}{16}\right) - 3(2) + \frac{5}{2}$

$= \frac{18}{4} - 6 + \frac{5}{2}$

$= \frac{9}{2} + \frac{5}{2} - 6 = \frac{14}{2} - 6 = 7 - 6 = 1$

$\boxed{4(\sin^4 60° + \cos^4 30°) - 3(\tan^2 60° - \tan^2 45°) + 5\cos^2 45° = 1}$

10. $(\operatorname{cosec}^2 45° \cdot \sec^2 30°)(\sin^2 30° + 4\cot^2 45° - \sec^2 60°)$

solution:- we know that,

$\operatorname{cosec} 45° = \sqrt{2}$, $\sec 30° = \frac{2}{\sqrt{3}}$

$\sin 30° = 1/2$, $\cot 45° = 1$, $\sec 60° = 2$

Now, $(\operatorname{cosec}^2 45° \cdot \sec^2 30°)(\sin^2 30° + 4\cot^2 45° - \sec^2 60°)$

$= \left[(\sqrt{2})^2 \left(\frac{2}{\sqrt{3}}\right)^2\right]\left[(1/2)^2 + 4(1)^2 - (2)^2\right]$

$= [2(4/3)][1/4 + 4 - 4]$

$= \left(\frac{8}{3}\right)\left(\frac{1}{4}\right)$

$= \frac{2}{3}$

11. $\operatorname{cosec}^3 30° \cdot \cos 60° \cdot \tan^3 45° \cdot \sin^2 90° \sec^2 45° \cdot \cot 30°$

soln → we all know that,

$\operatorname{cosec} 30° = 2$, $\cos 60° = 1/2$, $\tan 45° = 1$

$\sin 90° = 1$, $\sec 45° = \sqrt{2}$, $\cot 30° = \sqrt{3}$

Now,

$\operatorname{cosec}^3 30° \cdot \cos 60° \cdot \tan^3 45° \cdot \sin^2 90° \sec^2 45° \cot 30°$

$= (2)^3 (1/2) (1)^3 (1)^2 (\sqrt{2})^2 (\sqrt{3})$

$= \left(8 \times \frac{1}{2}\right)(2\sqrt{3})$

$= 8\sqrt{3}$

12. $\cot^2 30° - 2\cos^2 60° - (3/4)\sec^2 45° - 4\sec^2 30°$

solution:– $\cot 30° = \sqrt{3}$, $\cos 60° = 1/2$, $\sec 45° = \sqrt{2}$, $\sec 30° = \frac{2}{\sqrt{3}}$

Now, $\cot^2 30° - 2\cos^2 60° - 3/4 \sec^2 45° - 4\sec^2 30°$

$= (\sqrt{3})^2 - 2(1/2)^2 - 3/4(\sqrt{2})^2 - 4(2/\sqrt{3})^2$

$= 3 - 1/2 - 3/2 - 16/3$

$= 3 - 2 - 16/3$

$= 1 - 16/3$

$= \frac{-13}{3}$

13. $(\cos 0° + \sin 45° + \sin 30°)(\sin 90° - \cos 45° + \cos 60°)$

solution:– $\cos 0° = 1$, $\sin 45° = 1/\sqrt{2}$, $\sin 30° = 1/2$

$\sin 90° = 1$, $\cos 45° = 1/\sqrt{2}$, $\cos 60° = 1/2$

Now, $(\cos 0° + \sin 45° + \sin 30°)(\sin 90° - \cos 45° + \cos 60°)$

$= (1 + 1/\sqrt{2} + 1/2)(1 - 1/\sqrt{2} + 1/2)$

$= (3/2 + 1/\sqrt{2})(3/2 - 1/\sqrt{2})$

$= \left(\frac{3}{2}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2$ $\quad \because (a-b)(a+b) = a^2 - b^2$

$= \frac{9}{4} - \frac{1}{2} = \frac{7}{4}$

14. $\dfrac{\sin 30^\circ - \sin 90^\circ + 2\cos 0^\circ}{\tan 30^\circ \cdot \tan 60^\circ}$

Solution:- we know that,

$\sin 30^\circ = 1/2$, $\sin 90^\circ = 1$, $\cos 0^\circ = 1$

$\tan 30^\circ = 1/\sqrt{3}$, $\tan 60^\circ = \sqrt{3}$

Now, $\dfrac{\sin 30^\circ - \sin 90^\circ + 2\cos 0^\circ}{\tan 30^\circ \cdot \tan 60^\circ}$

$= \dfrac{(1/2) - (1) + 2}{(1/\sqrt{3}) \cdot \sqrt{3}}$

$= 1/2 + 1$

$= 3/2$.

15. $\dfrac{4}{\cot^2 30^\circ} + \dfrac{1}{\sin^2 60^\circ} - \cos^2 45^\circ$

Solution:- $\dfrac{4}{\cot^2 30^\circ} + \dfrac{1}{\sin^2 60^\circ} - \cos^2 45^\circ$

we know that, $\cot 30^\circ = \sqrt{3}$, $\sin 60^\circ = \sqrt{3}/2$, $\cos 45^\circ = 1/\sqrt{2}$

Now, $\dfrac{4}{\cot^2 30^\circ} + \dfrac{1}{\sin^2 60^\circ} - \cos^2 45^\circ$

$= \dfrac{4}{(\sqrt{3})^2} + \dfrac{1}{(\sqrt{3}/2)^2} - \left(\dfrac{1}{\sqrt{2}}\right)^2$

$= \dfrac{4}{3} + \dfrac{4}{3} - \dfrac{1}{2}$

$= \dfrac{8}{3} - \dfrac{1}{2} = \dfrac{16-3}{6}$

$= \dfrac{13}{6}$

16. $4(\sin^4 30^\circ + \cos^2 60^\circ) - 3(\cos^2 45^\circ - \sin^2 90^\circ) - \sin^2 60^\circ$

solution:- We all know that,

$\sin 30^\circ = 1/2$, $\cos 60^\circ = 1/2$, $\cos 45^\circ = 1/\sqrt{2}$, $\sin 90^\circ = 1$, $\sin 60^\circ = \sqrt{3}/2$

Now,

$4(\sin^4 30^\circ + \cos^2 60^\circ) - 3(\cos^2 45^\circ - \sin^2 90^\circ) - \sin^2 60^\circ$

$= 4\left[\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^2\right] - 3\left[\left(\frac{1}{\sqrt{2}}\right)^2 - (1)^2\right] - \left(\sqrt{3}/2\right)^2$

$= 4\left[\frac{1}{16} + \frac{1}{4}\right] - 3\left[\frac{1}{2} - 1\right] - 3/4$

$= \frac{1}{4} + 1 - \frac{3}{2} + 3 - \frac{3}{4}$

$= \frac{-1}{2} - \frac{3}{2} + 4 \quad = -2 + 4$

$= 2$

17. $\dfrac{\tan^2 60^\circ + 4\cos^2 45^\circ + 3\sec^3 30^\circ + 5\cos^2 90^\circ}{\operatorname{cosec} 30^\circ + \sec 60^\circ - \cot^2 30^\circ}$

solution:- we know that,

$\tan 60^\circ = \sqrt{3}$, $\cos 45^\circ = 1/\sqrt{2}$, $\sec 30^\circ = \frac{2}{\sqrt{3}}$, $\cos 90 = 0$

Now, $\dfrac{\tan^2 60^\circ + 4\cos^2 45^\circ + 3\sec^3 30^\circ + 5\cos^2 90^\circ}{\operatorname{cosec} 30^\circ + \sec 60^\circ - \cot^2 30^\circ}$

$= \dfrac{(\sqrt{3})^2 + 4(1/\sqrt{2})^2 + 3(2/\sqrt{3})^2 + 5(0)}{2 + 2 - (\sqrt{3})^2}$

$= 3 + 2 + 4$

$= 9$

18. $\frac{\sin 30^\circ}{\sin 45^\circ} + \frac{\tan 45^\circ}{\sec 60^\circ} - \frac{\sin 60^\circ}{\cot 45^\circ} - \frac{\cos 30^\circ}{\sin 90^\circ}$

solution:- we all know that,

$\sin 30^\circ = 1/2$, $\tan 45^\circ = 1$, $\sin 60^\circ = \sqrt{3}/2$

$\cos 30^\circ = \sqrt{3}/2$, $\sin 45^\circ = 1/\sqrt{2}$, $\sec 60^\circ = 2$

$\cot 45^\circ = 1$, $\sin 90^\circ = 1$.

Now, $= \frac{\sin 30^\circ}{\sin 45^\circ} + \frac{\tan 45^\circ}{\sec 60^\circ} - \frac{\sin 60^\circ}{\cot 45^\circ} - \frac{\cos 30^\circ}{\sin 90^\circ}$

$= \frac{1/2}{1/\sqrt{2}} + \frac{1}{2} - \frac{\sqrt{3}/2}{1} - \frac{\sqrt{3}/2}{1}$

$= \frac{\sqrt{2}}{2} + \frac{1}{2} - \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} = \frac{(\sqrt{2}+1-2\sqrt{3})}{2}$

$= \frac{(\sqrt{2}+1-2\sqrt{3})}{2}$

19. $\frac{\tan 45^\circ}{\operatorname{cosec} 30^\circ} + \frac{\sec 60^\circ}{\cot 45^\circ} - \frac{5 \sin 90^\circ}{2 \cos 0^\circ}$

solution:- We all know that,

$\tan 45^\circ = 1$, $\sec 60^\circ = 2$, $\sin 90^\circ = 1$

$\operatorname{cosec} 30^\circ = 2$, $\cot 45^\circ = 1$, $\cos 0^\circ = 1$

Now, $\frac{\tan 45^\circ}{\operatorname{cosec} 30^\circ} + \frac{\sec 60^\circ}{\cot 45^\circ} - \frac{5 \sin 90^\circ}{2 \cos 0^\circ}$

$= \frac{1}{2} + \frac{2}{1} - \frac{5}{2}$

$= \frac{5}{2} - \frac{5}{2}$

$= 0$

20. Find the value of x in the following:

$2\sin 3x = \sqrt{3}$

Solution:- $2\sin 3x = \sqrt{3}$

$\sin 3x = \frac{\sqrt{3}}{2}$ $\because \sin 60^\circ = \frac{\sqrt{3}}{2}$

$\sin 3x = \sin 60^\circ$

$3x = 60^\circ$

$\boxed{x = 20^\circ}$

Exercise 5.3

1. Evaluate the following:

1) $\sin 20^\circ / \cos 70^\circ$

$\rightarrow$

$\sin 20^\circ = \sin(90^\circ - 70^\circ)$. $\because \sin(90-\theta) = \cos\theta$

$\boxed{\sin 20^\circ = \cos 70^\circ}$

Here, $\frac{\sin 20^\circ}{\cos 70^\circ} = \frac{\cos 70^\circ}{\cos 70^\circ} = 1$

2) $\cos 19^\circ / \sin 71^\circ$

$\rightarrow$

$\cos 19^\circ = \cos(90^\circ - 71^\circ)$ $\because \cos(90-\theta) = \sin\theta$

$\boxed{\cos 19^\circ = \cos 71^\circ}$

Here, $\frac{\cos 19^\circ}{\sin 71^\circ} = \frac{\cos 71^\circ}{\cos 71^\circ} = 1$

iii) $\frac{\sin 21^\circ}{\cos 69^\circ}$

$\rightarrow$ $\sin 21^\circ = \sin(90^\circ - 69^\circ)$ $\quad \because \sin(90-\theta) = \cos\theta$

$\boxed{\sin 21^\circ = \cos 69^\circ}$

Here, $\frac{\sin 21^\circ}{\cos 69^\circ} = \frac{\cos 69^\circ}{\cos 69^\circ} = 1$

iv) $\tan 10^\circ / \cot 80^\circ$

$\rightarrow$ $\tan 10^\circ = \tan(90^\circ - 80^\circ)$ $\quad \because \tan(90-\theta) = \cot\theta$

$\boxed{\tan 10^\circ = \cot 80^\circ}$

Here, $\frac{\tan 10^\circ}{\cot 80^\circ} = \frac{\cot 80^\circ}{\cot 80^\circ} = 1$

v) $\frac{\sec 11^\circ}{\operatorname{cosec} 79^\circ}$

$\rightarrow$ $\sec 11^\circ = \sec(90^\circ - 79^\circ)$ $\quad \because \sec(90-\theta) = \operatorname{cosec}\theta$

$\boxed{\sec 11^\circ = \operatorname{cosec} 79^\circ}$

Here, $\frac{\sec 11^\circ}{\operatorname{cosec} 79^\circ} = \frac{\operatorname{cosec} 79^\circ}{\operatorname{cosec} 79^\circ} = 1$

2. Evaluate the following:

i) $\left(\frac{\sin 49^\circ}{\cos 41^\circ}\right)^2 + \left(\frac{\cos 41^\circ}{\sin 49^\circ}\right)^2$

$\rightarrow$ $\left(\frac{\sin 49^\circ}{\cos 41^\circ}\right)^2 + \left(\frac{\cos 41^\circ}{\sin 49^\circ}\right)^2$

$= \left[\frac{\sin(90^\circ - 41^\circ)}{\cos 41^\circ}\right]^2 + \left[\frac{\cos(90^\circ - 49^\circ)}{\sin 49^\circ}\right]^2$

But $\sin(90-\theta) = \cos\theta$ and $\cos(90-\theta) = \sin\theta$

$= \left(\frac{\cos 41^\circ}{\cos 41^\circ}\right)^2 + \left(\frac{\sin 49^\circ}{\sin 49^\circ}\right)^2 = 1^2 + 1^2 = 2$

ii) $\cos 48^\circ - \sin 42^\circ$

$\longrightarrow$ $\cos(90^\circ - 42^\circ) = \sin 42^\circ$ $\quad \because \cos(90^\circ - 42^\circ) = \sin 42^\circ$

$\cos(90 - \theta) = \sin\theta$

Now, $\cos 48^\circ - \sin 42^\circ = \sin 42^\circ - \sin 42^\circ = 0$

$$\boxed{\cos 48^\circ - \sin 42^\circ = 0}$$

iii) $\dfrac{\cot 40^\circ}{\tan 50^\circ} - \dfrac{1}{2}\left(\dfrac{\cos 35^\circ}{\sin 55^\circ}\right)$

$\longrightarrow$

$\cot(90 - \theta) = \tan\theta$

Here, $\cot(90^\circ - 50^\circ) = \tan 50^\circ$

and $\cos(90 - \theta) = \sin\theta \Rightarrow \cos(90^\circ - 55^\circ) = \sin 55^\circ$

Now, $\dfrac{\cot 40^\circ}{\tan 50^\circ} - \dfrac{1}{2}\left(\dfrac{\cos 35^\circ}{\sin 55^\circ}\right)$

$= \dfrac{\tan 50^\circ}{\tan 50^\circ} - \dfrac{1}{2}\dfrac{\sin 55^\circ}{\sin 55^\circ}$

$= 1 - \dfrac{1}{2}(1) = 1/2$

iv) $\left(\dfrac{\sin 27^\circ}{\cos 63^\circ}\right)^2 - \left(\dfrac{\cos 63^\circ}{\sin 27^\circ}\right)^2$

$\longrightarrow$ Here, $\sin(90^\circ - 63^\circ) = \cos 63^\circ$ $\quad \because \sin(90 - \theta) = \cos\theta$

$\sin 27^\circ = \sin(90^\circ - 63^\circ) = \cos 63^\circ$

And, $\cos 63^\circ = \cos(90^\circ - 27^\circ) = \sin 27^\circ$ $\quad \because \cos(90 - \theta) = \sin\theta$

Now, $\left(\dfrac{\sin 27^\circ}{\cos 63^\circ}\right)^2 - \left(\dfrac{\cos 63^\circ}{\sin 27^\circ}\right)^2$

$= \left(\dfrac{\cos 63^\circ}{\cos 63^\circ}\right)^2 - \left(\dfrac{\sin 27^\circ}{\sin 27^\circ}\right)^2$

$= 1 - 1$

$= 0$

v) $\frac{\tan 35^\circ}{\cot 55^\circ} + \frac{\cot 78^\circ}{\tan 12^\circ} - 1$

Solution:- we already know that,

$\tan 35^\circ = \tan(90^\circ - 55^\circ) = \cot 55^\circ$ $\because \tan(90-\theta) = \cot\theta$

$\cot 78^\circ = \cot(90^\circ - 12^\circ) = \tan 12^\circ$ $\because \cot(90^\circ - 12^\circ) = \tan 12^\circ$

Now, $\frac{\tan 35^\circ}{\cot 55^\circ} + \frac{\cot 78^\circ}{\tan 12^\circ} - 1$

$= \frac{\cot 55^\circ}{\cot 55^\circ} + \frac{\tan 12^\circ}{\tan 12^\circ} - 1$

$= 1 + 1 - 1$

$= 1$

vi) $\frac{\sec 70^\circ}{\csc 20^\circ} + \frac{\sin 59^\circ}{\cos 31^\circ}$

$\longrightarrow$

$\sec 70^\circ = \sec(90^\circ - 20^\circ) = \csc 20^\circ$ $\because \sec(90-\theta) = \csc\theta$

$\sin 59^\circ = \sin(90^\circ - 31^\circ) = \cos 31^\circ$ $\because \sin(90^\circ - 31^\circ) = \cos 31^\circ$

Now, $\frac{\sec 70^\circ}{\csc 20^\circ} + \frac{\sin 59^\circ}{\cos 31^\circ}$

$= \frac{\csc 20^\circ}{\csc 20^\circ} + \frac{\cos 31^\circ}{\cos 31^\circ}$

$= 1 + 1$

$= 2$

vii) $\csc 31^\circ - \sec 59^\circ$

$\longrightarrow$ $\csc 31^\circ = \csc(90 -^\circ 59^\circ) = \sec 59^\circ$ $\because \csc(90-\theta) = \sec\theta$

Here, $\csc 31^\circ - \sec 59^\circ$

$= \sec 59^\circ - \sec 59^\circ$

$= 0$

viii) $(\sin 72° + \cos 18°)(\sin 72° - \cos 18°)$

$\longrightarrow$ Here, $\sin 72° = \sin(90° - 18°) = \cos 18°$

$\because \sin(90 - \theta) = \cos\theta$

Now, $(\sin 72° + \cos 18°)(\sin 72° - \cos 18°)$

$= (\sin 72° + \cos 18°)(\sin 72° - \cos 18°)$

$= (\sin 72° + \cos 18°)(\cos 18° - \cos 18°)$

$= (\sin 72° + \cos 18°)(0)$

$= 0$

ix) $\sin 35° \cdot \sin 55° - \cos 35° \cdot \cos 55°$

$\longrightarrow$

$\sin 35° = \sin(90° - 55°) = \cos 55°$ $\quad \because \sin(90 - \theta) = \cos\theta$

and $\sin 55° = \sin(90° - 35°) = \cos 35°$ $\quad \because \sin(90 - \theta) = \cos\theta$

Now, $\sin 35° \cdot \sin 55° - \cos 35° \cdot \cos 55°$

$= \cos 55° \cdot \cos 35° - \cos 35° \cdot \cos 55°$

$= \cos 55° (\cos 35° - \cos 35°)$

$= \cos 55° (0)$

$= 0$

x) $\tan 48° \cdot \tan 23° \cdot \tan 42° \cdot \tan 67°$

solution $\rightarrow$ $\tan(90° - 42°) = \tan 48° = \cot 42°$ $\quad \because \tan(90 - \theta) = \cot\theta$

and $\tan(90° - 67°) = \tan 42° = \cot 67°$ $\quad \because \tan(90 - \theta) = \cot\theta$

Now, $\tan 48° \cdot \tan 23° \cdot \tan 42° \cdot \tan 67°$

$= \cot 42° \cdot \tan 23° \cdot \cot 67° \cdot \tan 67°$

$= (\cot 42° \times \tan 42°)(\tan 67° \times \cot 67°)$

$= 1 \times 1$ $\quad \because \tan(90° - \theta) = \cot\theta$

$= 1$

xi) $\sec 50^\circ \cdot \sin 40^\circ + \cos 40^\circ \cdot \operatorname{cosec} 50^\circ$

$\longrightarrow$ Now, $\sin(90-\theta) = \cos\theta$

$\Rightarrow \sin 40^\circ = \sin(90^\circ - 50^\circ) = \cos 50^\circ$

and $\cos(90-\theta) = \sin\theta$

$\Rightarrow \cos(90^\circ - 50^\circ) = \cos 40^\circ = \sin 50^\circ$

Here, $\sec 50^\circ \cdot \sin 40^\circ + \cos 40^\circ \cdot \operatorname{cosec} 50^\circ$

$= \sec 50^\circ \cdot \cos 50^\circ + \sin 50^\circ \cdot \operatorname{cosec} 50^\circ$

$= 1 + 1$

$= 2$

$\because \frac{1}{\cos\theta} = \sec\theta$

$\because \frac{1}{\sin\theta} = \operatorname{cosec}\theta$

3. Express each one of the following in terms of trigonometric ratios of angles lying between 0° and 45°.

i) $\sin 59^\circ + \cos 56^\circ$

$\longrightarrow$ $\sin(90-\theta) = \cos\theta$ and $\cos(90-\theta) = \sin\theta$

$\sin 59^\circ = \sin(90^\circ - 31^\circ) = \cos 31^\circ$ and $\cos 56^\circ = \cos(90^\circ - 34^\circ) = \sin 34^\circ$

Now, $\sin 59^\circ + \cos 56^\circ$

$= \cos 31^\circ + \sin 34^\circ$

ii) $\tan 65^\circ + \cot 49^\circ$

$\longrightarrow$ $\tan(90-\theta) = \cot\theta$ and $\cot(90-\theta) = \tan\theta$

$\tan 65^\circ = \tan(90^\circ - 25^\circ) = \cot 25^\circ$ and

$\cot 49^\circ = \cot(90^\circ - 41^\circ) = \tan 41^\circ$

Now, $\tan 65^\circ + \cot 49^\circ$

$= \cot 25^\circ + \tan 41^\circ$

iii) $\sec 76° + \operatorname{cosec} 52°$

$\rightarrow$ we know that,

$\sec(90-\theta) = \operatorname{cosec}\theta$ and $\operatorname{cosec}(90-\theta) = \sec\theta$

Here, $\sec 76° = \sec(90° - 14°) = \operatorname{cosec} 14°$

$\operatorname{cosec} 52° = \operatorname{cosec}(90° - 38°) = \sec 38°$

Now, $\sec 76° + \operatorname{cosec} 52°$

$= \operatorname{cosec} 14° + \sec 38°$

iv) $\cos 78° + \sec 78°$

$\rightarrow$ we know that,

$\cos(90-\theta) = \sin\theta$ and $\sec(90-\theta) = \operatorname{cosec}\theta$

Here, $\cos 78° = \cos(90° - 12°) = \sin 12°$ and

$\sec(90° - 12°) = \sec 78° = \operatorname{cosec} 12°$

Now, $\cos 78° + \sec 78°$

$= \sin 12° + \operatorname{cosec} 12°$

v) $\operatorname{cosec} 54° + \sin 72°$

$\rightarrow$ we know that,

$\operatorname{cosec}(90-\theta) = \sec\theta$ and $\sin(90-\theta) = \cos\theta$

Here, $\operatorname{cosec} 54° = \operatorname{cosec}(90° - 36°) = \sec 36°$

and $\sin 72° = \sin(90° - 18°) = \cos 18°$

Now, $\operatorname{cosec} 54° + \sin 72°$

$= \sec 36° + \cos 18°$

vi) $\cot 85^\circ + \cos 75^\circ$

→ we know that,

$\cot(90-\theta) = \tan\theta$ and $\cos(90-\theta) = \sin\theta$

Here, $\cot 85^\circ = \cot(90^\circ - 5^\circ) = \tan 5^\circ$

and $\cos 75^\circ = \cos(90^\circ - 15^\circ) = \sin 15^\circ$

Now, $\cot 85^\circ + \cos 75^\circ$

$= \tan 5^\circ + \sin 15^\circ$

4. Express $\cos 75^\circ + \cot 75^\circ$ in terms of angles between 0° & 30°.

→ Given that, $\cos 75^\circ + \cot 75^\circ$

we know that, $\cos(90-\theta) = \sin\theta$ and

$\sin(90-\theta) = \cos\theta$

$\cot(90-\theta) = \tan\theta$

$\cos 75^\circ + \cot 75^\circ$

$\Rightarrow \cos(90-15)^\circ + \cot(90-15)^\circ$

$\Rightarrow \boxed{\sin 15^\circ + \tan 15^\circ = \cos 75^\circ + \cot 75^\circ}$

5. If $\sin 3A = \cos(A - 26^\circ)$, where $3A$ is an acute angle, find the value of A.

→ Here, given that $\sin 3A = \cos(A - 26^\circ)$

we know that, $\cos(90-\theta) = \sin\theta$

$\Rightarrow \sin 3A = \sin[90^\circ - (A - 26^\circ)]$

$\Rightarrow 3A = 90^\circ - (A - 26^\circ)$

$3A + A - 26^\circ = 90^\circ$

$4A = 90^\circ + 26^\circ$

$4A = 116^\circ$

$\boxed{A = 29^\circ}$

6. If A, B, C are the interior angles of a triangle ABC, prove that i) $\tan[(C+A)/2] = \cot(B/2)$

ii) $\sin[(B+C)/2] = \cos(A/2)$

→ we already know that, the sum of three angles of a triangle is $180°$. ∴ $A+B+C = 180°$

$C+A = 180° - B$

$(C+A)/2 = 90° - B/2$ —— ①

And $B+C = 180° - A$

$(B+C)/2 = 90° - A/2$ —— ②

Now, i) $\tan[(C+A)/2] = \cot(B/2)$

$\tan(90° - B/2) = \cot(B/2)$ ∵ by ①

$\cot B/2 = \cot(B/2)$ ∵ $\tan(90-\theta) = \cot\theta$

ii) $\sin[(B+C)/2] = \cos(A/2)$

$\sin(90° - A/2) = \cos(A/2)$ ∵ by ②

$\cos(A/2) = \cos(A/2)$ ∵ $\sin(90-\theta) = \cos\theta$

7. Prove that:

i) $\tan 20° . \tan 35° . \tan 45° . \tan 55° . \tan 70° = 1$

→ L.H.S. $= \tan 20° . \tan 35° . \tan 45° . \tan 55° . \tan 70°$

$= \tan(90-70)° . \tan(90-55)° . \tan 55° . \tan 70°$

$= \cot 70° . \cot 55° . \tan 55° . \tan 70°$

$= (\cot 70° . \tan 70°)(\cot 55° . \tan 55°)$ ∵ $\tan(90-\theta) = \cot\theta$

∵ $\tan\theta \times \cot\theta = 1$

$= 1 \times 1$

$= 1$

$=$ R.H.S. Hence proved

ii) $\sin 42^\circ \sec 48^\circ + \cos 48^\circ \operatorname{cosec} 42^\circ = 2$

→ L.H.S. = [illegible]

$=$ [illegible] $+$ [illegible] 2

$=$ [illegible] $+$ [illegible]

$\sin 42^\circ \cdot \sec 48^\circ + \cos 48^\circ \cdot \operatorname{cosec} 42^\circ = 2$

→ Here,

L.H.S. $= \sin 42^\circ \cdot \sec 48^\circ + \cos 48^\circ \cdot \operatorname{cosec} 42^\circ$

$= \sin 42^\circ \cdot \sec(90-42)^\circ + \cos 48^\circ \cdot \operatorname{cosec}(90-48)^\circ$

$= \sin 42^\circ \cdot \cos 42^\circ + \cos 48^\circ \cdot \sec 48^\circ$

$= [\sin 42^\circ \times \operatorname{cosec} 42^\circ] + [\cos 48^\circ \times \sec 48^\circ]$

$\because \sec(90-\theta) = \operatorname{cosec}\theta$ and $\operatorname{cosec}(90-\theta) = \sec\theta$

also, $\sin\theta \times \operatorname{cosec}\theta = 1$, $\cos\theta \times \sec\theta = 1$

L.H.S. $= 1 + 1 = 2 =$ R.H.S. Hence proved.

iii) $\dfrac{\sin 70^\circ}{\cos 20^\circ} + \dfrac{\operatorname{cosec} 20^\circ}{\sec 70^\circ} - 2\cos 70^\circ \cdot \operatorname{cosec} 20^\circ = 0$

→ L.H.S. $= \dfrac{\sin 70^\circ}{\cos 20^\circ} + \dfrac{\operatorname{cosec} 20^\circ}{\sec 70^\circ} - 2\cos 70^\circ \cdot \operatorname{cosec} 20^\circ$

$= \dfrac{\sin(90-20)}{\cos 20^\circ} + \dfrac{\operatorname{cosec}(90-70)}{\sec 70^\circ} - 2\cos(90-20) \cdot \operatorname{cosec} 20^\circ$

$= \dfrac{\cos 20^\circ}{\cos 20^\circ} + \dfrac{\sec 70^\circ}{\sec 70^\circ} - 2\,\cancel{\sec}\,\sin 20^\circ \cdot \operatorname{cosec} 20^\circ$

$= 1 + 1 - 2 \times 1$

$\because \sin(90-\theta) = \cos\theta$

$\operatorname{cosec}(90-\theta) = \sec\theta$

$\sin\theta \times \operatorname{cosec}\theta = 1$

$= 2 - 2$

$= 0$

$=$ R.H.S.

Hence proved

iv) $\frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \cdot \csc 31^\circ$

$\longrightarrow$ L.H.S $= \frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \csc 31^\circ$

$= \frac{\cos(90-80)}{\sin 10} + \cos 59 \cdot \csc 31$

$= \frac{\sin 10^\circ}{\sin 10^\circ} + \cos 59 \cdot \csc(90-59)$

$= 1 + \cos 59 \cdot \sec(59)$

$= 1+1$

$= 2$

$=$ R.H.S.

$\because \cos(90-\theta) = \sin\theta$

$\csc(90-\theta) = \sec\theta$

$\csc\theta \times \sin\theta = 1$

$\cos\theta \times \sec\theta = 1$

Hence proved.

8. Prove that followings:

i) $\sin\theta \cdot \sin(90-\theta) - \cos\theta \cdot \cos(90-\theta) = 0$

$\longrightarrow$ LHS $= \sin\theta \cdot \sin(90-\theta) - \cos\theta \cdot \cos(90-\theta)$

$= \sin\theta \cdot \cos\theta - \cos\theta \cdot \sin\theta$

$= \sin\theta\,(\cos\theta - \cos\theta)$

$= 0$

$=$ R.H.S. Hence proved

$\because \sin(90-\theta) = \cos\theta$

$\cos(90-\theta) = \sin\theta$

ii) $\frac{\cos(90^\circ-\theta) \cdot \sec(90^\circ-\theta) \tan\theta}{\csc(90-\theta) \cdot \sin(90^\circ-\theta) \cdot \cot(90-\theta)} + \frac{\tan(90-\theta)}{\cot\theta} = 2$

$\longrightarrow$ L.H.S. $= \frac{\cos(90^\circ-\theta) \cdot \sec(90^\circ-\theta) \cdot \tan\theta}{\csc(90-\theta) \cdot \sin(90-\theta) \cdot \cot(90-\theta)} + \frac{\tan(90-\theta)}{\cot\theta}$

$= \frac{\sin\theta \cdot \csc\theta \cdot \tan\theta}{\sec\theta \cdot \cos\theta \cdot \tan\theta} + \frac{\cot\theta}{\cot\theta}$

$= \frac{\tan\theta}{\tan\theta} + 1 = 1+1 = 2 =$ R.H.S. Hence proved.

iii) $\dfrac{\tan(90^\circ - A)\cdot \cot A}{\operatorname{cosec}^2 A} - \cos^2 A = 0$

$\longrightarrow$

$$\text{L.H.S.} = \frac{\tan(90^\circ - A)\cdot \cot A}{\operatorname{cosec}^2 A} - \cos^2 A$$

$$= \frac{\cot A \cdot \cot A}{\operatorname{cosec}^2 A} - \cos^2 A \qquad \because \tan(90-\theta) = \cot\theta$$

$$= \frac{\cot^2 A}{\operatorname{cosec}^2 A} - \cos^2 A$$

$$= \sin^2 A /$$

$$= \frac{(\cos^2 A / \sin^2 A)}{(1/\sin^2 A)} - \cos^2 A \qquad \because \operatorname{cosec} A = \frac{1}{\sin A}$$

$$= \cos^2 A - \cos^2 A \qquad \because \cot A = \frac{\cos A}{\sin A}$$

$$= 0$$

$= \text{R.H.S.}$ Hence proved

iv) $\dfrac{\cos(90^\circ - A)\cdot \sin(90^\circ - A)}{\tan(90^\circ - A)} = \sin^2 A$

$\longrightarrow$

$$\text{L.H.S.} = \frac{\cos(90^\circ - A)\cdot \sin(90^\circ - A)}{\tan(90^\circ - A)}$$

$$= \frac{\sin A \cdot \cos A}{\cot A}$$

$\because \cos(90-\theta) = \sin\theta$

$\because \sin(90-\theta) = \cos\theta$

$$= \frac{\sin A \cdot \cos A}{\cos A / \sin A}$$

$\because \tan(90-\theta) = \cot\theta$

$\because \cot\theta = \cos\theta / \sin\theta$

$$= \sin^2 A$$

$= \text{R.H.S.}$ Hence proved.

v) $\sin(50^\circ+\theta) - \cos(40^\circ-\theta) + \tan 1^\circ \cdot \tan 10^\circ \cdot \tan 20^\circ \cdot \tan 70^\circ \cdot \tan 80^\circ \cdot \tan 89^\circ = 1$

$\longrightarrow$ Here,

LHS $= \sin(50^\circ+\theta) - \cos(40^\circ-\theta) + \tan 1^\circ \cdot \tan 10^\circ \cdot \tan 20^\circ \cdot \tan 70^\circ \cdot \tan 80^\circ \cdot \tan 89^\circ$

$= \sin[90-(40^\circ-\theta)] - \cos(40^\circ-\theta) + \tan(90-89^\circ) \cdot \tan(90-80^\circ) \cdot \tan(90-70)^\circ \cdot \tan 70^\circ \cdot \tan 80^\circ \cdot \tan 89^\circ$

$= \cos(40^\circ-\theta) - \cos(40^\circ-\theta) + (\cot 89^\circ \cdot \cot 80^\circ \cdot \cot 70^\circ)(\tan 70^\circ \cdot \tan 80^\circ \cdot \tan 89^\circ)$

$= 0 + (\cot 89 \times \tan 89)(\cot 80 \times \tan 80)(\cot 70 \times \tan 70)$

$= 0 + 1 \times 1 \times 1$

$= 1$

$=$ R.H.S.

Hence proved

$\because \sin(90-\theta) = \cos\theta$

$\tan(90-\theta) = \cot\theta$

$\cot\theta \times \tan\theta = 1$