Chapter 2: Polynomials

Exercise 2.1

Q.1.) Find the zeroes of each of the following quadratic polynomials and verify the relationship between the zeroes of their coefficients:

i) $f(x) = x^2 - 2x - 8$

→ Given that, $f(x) = x^2 - 2x - 8$

put $f(x)=0$, to get zeroes-

$\Rightarrow f(x) = 0$

$x^2 - 2x - 8 = 0$

$x^2 - 4x + 2x - 8 = 0$

$x(x-4) + 2(x-4) = 0$

$(x-4)(x+2) = 0$

$x = 4$ or $x = -2$

Here, the zeroes of given quadratic polynomial are 4 and -2.

To verify:

Sum of zeros = coefficient of x / coefficient of x^2

$4 + (-2) = -(-2)/1$

$2 = 2$

Product of roots = constant / coefficient of x^2

$4 \times (-2) = +(-8)/1$

$-8 = -8$

Thus, the relationship betn zeros & their coefficients is verified.

ii) $f(x) = g(s) = 4s^2 - 4s + 1$

→ Given polynomial is $g(s) = 4s^2 - 4s + 1$

To find zero of polynomial, we take $g(s) = 0$

$$\Rightarrow 4s^2 - 4s + 1 = 0$$

$$4s^2 - 2s - 2s + 1 = 0$$

$$2s(2s-1) - 1(2s-1) = 0$$

$$(2s-1)(2s-1) = 0$$

$$\boxed{s = 1/2} \quad \boxed{s = 1/2}$$

The zeros of the quadratic equation are $1/2$ & $1/2$.

To verify:

Sum of zeros = $-$coefficient of s / coefficient of s^2

$$1/2 + 1/2 = -(-4)/4$$

$$1 = 1$$

Product of zeros = constant / coefficient of s^2

$$1/2 \times 1/2 = 1/4$$

$$1/4 = 1/4$$

Thus, the relationship betn zeros & their coefficients is verified.

iii) $h(t) = t^2 - 15$

→ Given quadratic polynomial is $h(t) = t^2 - 15$.

To find the zeros of polynomial, we take $h(t) = 0$.

$$\Rightarrow t^2 - 15 = 0$$

$$t^2 = 15$$

$$\boxed{t = \pm\sqrt{15}}$$

Here, the zeros of given polynomial are $\sqrt{15}$ and $-\sqrt{15}$.

To verify:

Sum of zeros = − coefficient of t / coefficient of t^2

$(\sqrt{15})+(-\sqrt{15}) = -0/1$

$0=0$

Product of roots = constant / coefficient of t^2

$\sqrt{15}\times(-\sqrt{15}) = -15/1$

$-15=-15$

Thus, the relationship between zeros and their coefficient is verified.

iv) $f(x)=6x^2-3-7x$

→ Given quadratic equation is $f(x)=6x^2-3-7x$

To find zeros, we take $f(x)=0$

$\Rightarrow 6x^2-3-7x=0$

$6x^2-7x-3=0$

$6x^2-9x+2x-3=0$

$3x(2x-3)+1(2x-3)=0$

$(2x-3)(3x+1)=0$

$x=3/2$ and $x=-1/3$

Here, the zeros of quadratic eqn are $3/2$ and $-1/3$.

To verify:

sum of zeros = − coefficient of x / coefficient of x^2

$3/2+(-1/3) = -(-7)/6$

$7/6=7/6$

Product of roots = constant / coefficient of x^2

$3/2\times(-1/3)=(-3)/6$

$-1/2=-1/2$

Thus, the relationship betn zeros & their coefficients is verified.

v) $p(x) = x^2 + 2\sqrt{2}x - 6$

→ Given, quadratic polynomial is $x^2+2\sqrt{2}x-6 = p(x)$.

To find zeros, we take $p(x)=0$

$$\Rightarrow x^2+2\sqrt{2}x-6=0$$

$$x^2+3\sqrt{2}x-\sqrt{2}x-6=0$$

$$x(x+3\sqrt{2})-\sqrt{2}(x+3\sqrt{2})=0$$

$$(x+3\sqrt{2})(x-\sqrt{2})=0$$

$$\boxed{x=-3\sqrt{2}} \text{ and } \boxed{x=\sqrt{2}}$$

Thus, here the roots of given quadratic equn are $-3\sqrt{2}$ & $\sqrt{2}$.

To verify:

Sum of zeros = $-$ coefficient of x / coefficient of x^2

$$\sqrt{2}+(-3\sqrt{2}) = -(2\sqrt{2})/1$$

$$-2\sqrt{2} = -2\sqrt{2}$$

Product of roots = constant / coefficient of x^2

$$\sqrt{2}\times(-3\sqrt{2}) = (-6)/2\sqrt{2}$$

$$-3\times 2 = -6/1$$

$$-6=-6$$

Hence, the relationship between the zeros & their coefficients is verified.

vi) $q(x) = \sqrt{3}x^2+10x+7\sqrt{3}$

→ Given, quadratic polynomial is $q(x)=\sqrt{3}x^2+10x+7\sqrt{3}$

To find the roots of polynomial, we take $q(x)=0$.

$$\sqrt{3}x^2+10x+7\sqrt{3}=0$$

$$\sqrt{3}x^2+3x+7x+7\sqrt{3}=0$$

$$\sqrt{3}x(x+\sqrt{3})+7(x+\sqrt{3})=0$$

$$(x+\sqrt{3})\cdot(\sqrt{3}x+7)=0$$

$$x=-\sqrt{3} \text{ and } x=-7/\sqrt{3}$$

Thus, the zeros of given polynomial are $-\sqrt{3}$ and $-7/\sqrt{3}$

To verify:

Sum of zeros = $-$coefficient of x / coefficient of x^2

$$-\sqrt{3}+(-7/\sqrt{3}) = -(10)/\sqrt{3}$$

$$(-3-7)/\sqrt{3} = -10/\sqrt{3}$$

$$-10/\sqrt{3} = -10/\sqrt{3}$$

Product of roots = constant / coefficient of x^2

$$(-\sqrt{3})\times(-7/\sqrt{3}) = (7\sqrt{3})/\sqrt{3}$$

$$7 = 7$$

Thus, the relationship betn zeros & their coefficients is verified.

vii) $f(x) = x^2 - (\sqrt{3}+1)x + \sqrt{3}$

→ Given, polynomial is $f(x) = x^2 - (\sqrt{3}+1)x + \sqrt{3}$

To find zeros, we take $f(x) = 0$

$$\Rightarrow x^2 - (\sqrt{3}+1)x + \sqrt{3} = 0$$

$$x^2 - \sqrt{3}x - x + \sqrt{3} = 0$$

$$x(x-\sqrt{3}) - 1(x-\sqrt{3}) = 0$$

$$(x-\sqrt{3})(x-1) = 0$$

$$x = \sqrt{3} \text{ \& } x = 1$$

Hence, the zeros of given quadratic poly. are $\sqrt{3}$ & 1.

To verify:

Sum of zeros = $-$coefficient of x / coefficient of x^2

$$\sqrt{3}+1 = -(-(\sqrt{3}+1))/1$$

$$\sqrt{3}+1 = \sqrt{3}+1$$

Product of roots = constant / coefficient of x^2

$$1\times\sqrt{3} = \sqrt{3}/1$$

$$\sqrt{3} = \sqrt{3}$$

Hence verified.

viii) $g(x) = a(x^2+1) - x(a^2+1)$

→ Given, polynomial is $g(x) = a(x^2+1) - x(a^2+1)$

To find zeros, take $g(x) = 0$

$\Rightarrow a(x^2+1) - x(a^2+1) = 0$

$ax^2 + a - a^2x - x = 0$

$ax(x-a) - 1(x-a) = 0$

$(x-a)(ax-1) = 0$

$\boxed{x=a}$ or $\boxed{x=1/a}$

Thus, the zeros of the quadratic eqn are a & $1/a$.

To verify:

Sum of zeros = − coefficient of x / coefficient of x^2

$a + 1/a = -(-(a^2+1))/a$

$(a^2+1)/a = (a^2+1)/a$

Product of roots = constant / coefficient of x^2

$a \times 1/a = a/a$

$1 = 1$

Thus, the relationship betn zeros & their coefficients is verified.

ix) $h(s) = 2s^2 - (1+2\sqrt{2})s + \sqrt{2}$

→ Given polynomial is $h(s) = 2s^2 - (1+2\sqrt{2})s + \sqrt{2}$

To find the zeros, take $h(s) = 0$

$\Rightarrow 2s^2 - (1+2\sqrt{2})s + \sqrt{2} = 0$

$2s^2 - 2\sqrt{2}s - s + \sqrt{2} = 0$

$2s(s-\sqrt{2}) - 1(s-\sqrt{2}) = 0$

$(2s-1)(s-\sqrt{2}) = 0$

$s = 1/2 \quad s = \sqrt{2}$

Thus, the zeros of the polynomial are $1/2$ and $\sqrt{2}$.

For verification:

Sum of zeros = - coefficent of s / coefficient of s^2

$\sqrt{2} + 1/2 = -(-(1+2\sqrt{2}))/2$

$(2\sqrt{2}+1)/2 = (2\sqrt{2}+1)/2$

Product of roots = constant / coefficient of s^2

$1/2 \times \sqrt{2} = \sqrt{2}/2$

$\sqrt{2}/2 = \sqrt{2}/2$

Thus, the relationship betn zeros & coefficient is verified.

x) $f(v) = v^2 + 4\sqrt{3}v - 15$

→ Given polynomial is $f(v) = v^2 + 4\sqrt{3}v - 15$

To find the zeros, put $f(v) = 0$

$\Rightarrow v^2 + 4\sqrt{3}v - 15 = 0$

$v^2 + 5\sqrt{3}v - \sqrt{3}v - 15 = 0$

$v(v + 5\sqrt{3}) - \sqrt{3}(v + 5/\sqrt{3}) = 0$

$(v - \sqrt{3})(v + 5\sqrt{3}) = 0$

$v = \sqrt{3}$ and $v = -5\sqrt{3}$

Thus, the roots of given polynomial are $\sqrt{3}$ and $-5\sqrt{3}$.

To verify:

Sum of zeros = - coefficient of v / coefficient of v^2

$\sqrt{3} + (-5\sqrt{3}) = -(4\sqrt{3})/1$

$-4\sqrt{3} = -4\sqrt{3}$

Product of roots = constant / coefficient of v^2

$\sqrt{3} \times (-5\sqrt{3}) = (-15)/1$

$-5 \times 3 = -15$

$-15 = -15$

Thus, the relationship betn zeros and their coefficients is verified.

xi) $p(y) = y^2 + (3\sqrt{5}/2)\, y - 5$

$\rightarrow$ Given polynomial is $p(y) = y^2 + (3\sqrt{5}/2)y - 5$

To find zeros of polynomial, take $p(y) = 0$

$$y^2 + (3\sqrt{5}/2)y - 5 = 0$$

$$y^2 - \sqrt{5}/2\,y + 2\sqrt{5}y - 5 = 0$$

$$y(y - \sqrt{5}/2) + 2\sqrt{5}(y - \sqrt{5}/2) = 0$$

$$(y + 2\sqrt{5})(y - \sqrt{5}/2) = 0$$

$$\boxed{y = \sqrt{5}/2} \text{ and } \boxed{y = -2\sqrt{5}}$$

Thus, the zeros of given polynomial are $\sqrt{5}/2$ & $-2\sqrt{5}$.

To verify:

Sum of zeros $= -$ coefficient of y / coefficient of y^2

$$\sqrt{5}/2 + (-2\sqrt{5}) = -(3\sqrt{5}/2)/1$$

$$-3\sqrt{5}/2 = -3\sqrt{5}/2$$

Product of roots = constant / coefficient of y^2

$$\sqrt{5}/2 \times (-2\sqrt{5}) = (-5)/1$$

$$-(\sqrt{5})^2 = -5$$

$$-5 = -5$$

Thus, the relationship betn zeros and their coefficients is verified.

xii) $q(y) = 7y^2 - (11/3)y - 2/3$

→ Given quadratic polynomial is $q(y) = 7y^2 - (11/3)y - 2/3$

To find zero, we take $q(y) = 0$

$$7y^2 - (11/3)y - 2/3 = 0$$

$$(21y^2 - 11y - 2)/3 = 0$$

$$21y^2 - 11y - 2 = 0$$

$$21y^2 - 14y + 3y - 2 = 0$$

$$7y(3y-2) - 1(3y+2) = 0$$

$$(3y-2)(7y+1) = 0$$

$$\boxed{y = 2/3} \text{ and } \boxed{y = -1/7}$$

Thus, the zeros of given polynomial are $2/3$ & $-1/7$.

To verify:

Sum of zeros = − coefficient of y / coefficient of y^2

$$2/3 + (-1/7) = -(-11/3)/7$$

$$-11/21 = -11/21$$

Product of roots = constant / coefficient of y^2

$$2/3 \times (-1/7) = (-2/3)/7$$

$$-2/21 = -2/21$$

Thus, the relationship betn zeros & their coefficients is verified.

Q.2) For each of the following, find a quadratic polynomial whose sum & product respectively of the zeros are as given. Also, find the zeros of these polyno. by factorization.

i) $-8/3$, $4/3$

→ A quadratic polynomial in the form of sum & product of zeros of polynomial is given as:

$$f(x) = x^2 + [-(\text{sum of zeros})]x + \text{product of roots}$$

Given that, sum $= -8/3$ and product $= 4/3$

Then, the required polynomial is

$$f(x) = x^2 + 8/3\, x + 4/3$$

To find zeros, we take $f(x) = 0$

$$\Rightarrow x^2 + 8/3x + 4/3 = 0$$

$$3x^2 + 8x + 4 = 0$$

$$3x(x+2) + 2(x+2) = 0$$

$$(x+2)(3x+2) = 0$$

$$x = -2 \text{ and } x = -2/3$$

The roots of the given polynomial are -2 and $-2/3$.

ii) $21/8$, $5/16$

→ A quadratic polynomial in the form of sum & product of zeros of polynomial is given as:

$$f(x) = x^2 - (\text{sum of zeros})x + \text{product of zeros}$$

$$f(x) = x^2 - (21/8)x + 5/16$$

To find roots, we take $f(x) = 0$

$$x^2 - 21/8x + 5/16 = 0$$

$$16x^2 - 42x + 5 = 0$$

$$16x^2 - 40x - 2x + 5 = 0$$

$$8x(2x-5) - 1(2x-5) = 0$$

$$(2x-5)(8x-1)=0$$
$$(2x-5)=0 \quad or \quad (8x-1)=0$$
Thus, $\boxed{x=5/2}$ & $\boxed{x=1/8}$

Thus, the zeros of given polynomial are 5/2 and 1/8.

iii) $-2\sqrt{3}, -9$

→ The quadratic polynomial in the form of sum & product of zeros is given by

$$f(x) = x^2 + (-\text{sum of zeros})\,x + \text{product of roots}$$
$$f(x) = x^2 - (-2\sqrt{3})x + (-9)$$
$$f(x) = x^2 + 2\sqrt{3}x - 9$$

To find zeros, we put $f(x)=0$

$$x^2 + 2\sqrt{3}x - 9 = 0$$
$$x^2 + 3\sqrt{3}x - \sqrt{3}x - 9 = 0$$
$$x(x+3\sqrt{3}) - \sqrt{3}(x+3\sqrt{3}) = 0$$
$$(x-\sqrt{3})(x+3\sqrt{3}) = 0$$
$$x-\sqrt{3}=0 \quad or \quad x+3\sqrt{3}=0$$
$$x=\sqrt{3} \quad or \quad x=-3\sqrt{3}$$

Thus, the roots of the given polynomial are $\sqrt{3}$ & $-3\sqrt{3}$.

iv) $-3/2\sqrt{5}, -1/2$

→ The quadratic polynomial in the form of sum & product of roots is given by,

$$f(x) = x^2 + (-\text{sum of zeros})\,x + (\text{product of roots})$$

$$f(x) = x^2 - (-3/2\sqrt{5})\,x + (-1/2)$$

$$f(x) = x^2 + 3/2\sqrt{5}\,x - 1/2$$

To find roots, we take $f(x) = 0$

$$x^2 + 3/2\sqrt{5}\,x - 1/2 = 0$$

$$2\sqrt{5}x^2 + 3x - \sqrt{5} = 0$$

$$2\sqrt{5}x^2 + 5x - 2x - \sqrt{5} = 0$$

$$\sqrt{5}x(2x + \sqrt{5}) - 1(2x + \sqrt{5}) = 0$$

$$(2x + \sqrt{5})(\sqrt{5}x - 1) = 0$$

$$(2x + \sqrt{5}) = 0 \text{ and } (\sqrt{5}x - 1) = 0$$

$$\boxed{x = -\sqrt{5}/2} \text{ and } \boxed{x = 1/\sqrt{5}}$$

Thus, the required two roots of poly. are $-\sqrt{5}/2$ or $1/\sqrt{5}$

Q. 3) If α and β are the zeros of the polynomial $f(x) = x^2 - 5x + 4$, find the value of $1/\alpha + 1/\beta - 2\alpha\beta$.

→ The given polynomial is $f(x) = x^2 - 5x + 4$

where $a = 1$, $b = -5$, $c = 4$

Given that, α and β are the zeros of poly.

Here, sum of roots $= \alpha + \beta = -b/a = -(-5)/1 = 5$

product of roots $= \alpha\beta = c/a = 4/1 = 4$

To find here, $1/\alpha + 1/\beta - 2\alpha\beta = [(\alpha+\beta)/\alpha\beta] - 2\alpha\beta$

$$\Rightarrow (5)/4 - 2(4) = 5/4 - 8 = -27/4$$

Thus, $\boxed{1/\alpha + 1/\beta - 2\alpha\beta = -27/4}$

Q.4.) If α and β are the zeros of quadratic polynomial $p(y) = 5y^2 - 7y + 1$, find the value of $1/\alpha + 1/\beta$.

Soln →

Given that, the polynomial $p(y) = 5y^2 - 7y + 1$

where, $a = 5$, $b = -7$, $c = 1$

Here, α & β are the roots of given polynomial.

Thus, sum of roots $= \alpha + \beta = -b/a = -(-7)/5 = 7/5$

product of roots $= \alpha\beta = c/a = 1/5$

Thus, here, $\frac{1}{\alpha} + \frac{1}{\beta} = (\alpha+\beta)/\alpha\beta$

$= (7/5)/(1/5)$

$$\boxed{\frac{1}{\alpha} + \frac{1}{\beta} = 7}$$

Q.5.) If α & β are the zeros of the quadratic polynomial $f(x) = x^2 - x - 4$, find the value of $1/\alpha + 1/\beta - \alpha\beta$.

→ The given quadratic polynomial is $f(x) = x^2 - x - 4$

where $a = 1$, $b = -1$, $c = -4$

Here, α & β are the roots of given polynomial.

Thus, sum of roots $= \alpha + \beta = -b/a = -(-1)/1 = 1$

product of roots $= \alpha\beta = c/a = -4/1 = -4$

Here, $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta = [(\alpha+\beta)/\alpha\beta] - \alpha\beta$

$= [(1)/(-4)] - (-4)$

$= -1/4 + 4$

$$\boxed{\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta = 15/4}$$

Q.6) If α and β are the zeros of the quadratic polynomial $f(x) = x^2+x-2$, find the values of $1/\alpha - 1/\beta$.

Soln → The given polynomial is $f(x) = x^2+x-2$

where, $a=1$, $b=1$, $c=-2$

Here, α and β are the two roots of given polynomial.

Sum of roots $= \alpha+\beta = -b/a = -(1)/1 = -1$

product of roots $= \alpha\beta = c/a = -2/1 = -2$

$$\text{Here, } \frac{1}{\alpha} - \frac{1}{\beta} = \frac{(\beta-\alpha)}{\alpha\beta}$$

$$= \frac{\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}}{\alpha\beta}$$

$$= \frac{\sqrt{1+8}}{2}$$

$$= \frac{\sqrt{9}}{2}$$

$$\boxed{\frac{1}{\alpha} - \frac{1}{\beta} = 3/2}$$

Q.7) If one of the zero of the quadratic polynomial $f(x) = 4x^2 - 8kx - 9$ is negative of the other, then find the value of k.

Soln → The given polynomial is $f(x) = 4x^2 - 8kx - 9$

where, $a=4$, $b=-8k$, $c=-9$

Given that, the roots are negative of each other.

Let us consider the roots will be α and $-\alpha$.

Thus, we can write

sum of roots $= \alpha - \alpha = -b/a = -(-8k)/1 = 8k = 0$

$$\boxed{k=0}$$

Q.8) If the sum of the zeros of the quadratic polynomial $f(t) = kt^2 + 2t + 3k$ is equal to their product, then find the value of k.

Soln:— The given polynomial is $f(t) = kt^2 + 2t + 3k$

where, $a = k$, $b = 2$, $c = 3k$

we know that,

Since, Sum of the roots = Product of roots

$$(-b/a) = (c/a)$$

$$-2/k = 3k/k$$

$$-2/k = 3$$

$$\therefore \boxed{k = -2/3}$$

Q.9. If α and β are the zeros of the quadratic polynomial $p(x) = 4x^2 - 5x - 1$, find the value of $\alpha^2\beta + \alpha\beta^2$.

Soln→ Given, the polynomial is $p(x) = 4x^2 - 5x - 1$

where $a = 4$, $b = -5$, $c = -1$

Here, sum of roots $= \alpha + \beta = -b/a = -(-5)/4 = 5/4$

product of roots $= \alpha\beta = c/a = -1/4$

Now, we find $\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha + \beta)$

$$= (-1/4)(5/4)$$

$$\boxed{\alpha^2\beta + \alpha\beta^2 = -5/16}$$

Q.10.) If α and β are the zeros of the quadratic polynomial $f(t)=t^2-4t+3$, find the value of $\alpha^4\beta^3+\beta^4\alpha^3$.

Sol^n → The given polynomial is $f(t)=t^2-4t+3$

where $a=1$, $b=-4$, $c=3$

Let α, β be the roots of given polynomial.

Now, sum of roots $= \alpha+\beta = -b/a = -(-4)/1 = 4$

product of roots $= \alpha\beta = c/a = 3/1 = 3$

Now, we will find

$$\alpha^4\beta^3+\alpha^3\beta^4 = (\alpha\beta)^3(\alpha+\beta)$$
$$= (3)^3(4)$$
$$= 27\times 4$$
$$\boxed{\alpha^4\beta^3+\alpha^3\beta^4 = 108}$$

Exercise 2.2

Q.1.) Verify that the numbers given alongside of the cubic polynomials below are their zeros. Also, verify the relationship betn the zeros and coefficients in each of the following cases:

i) $f(x)= 2x^3+x^2-5x+2$; $1/2$, 1, -2.

→ The given polynomial is $f(x)=2x^3+x^2-5x+2$

where $a=2$, $b=1$, $c=-5$, $d=2$

when $x=1/2$ ⇒ $f(1/2)=2(1/2)^3+(1/2)^2-5(1/2)+2$

$$= 1/4+1/4-5/2+2$$
$$= -2+2$$
$$\boxed{f(1/2)=0}$$

Thus, $x=1/2$ is the root of given polynomial.

when $x=1$,

$f(1)=2(1)^3+(1)^2-5(1)+2$

$=2+1-5+2$

$\boxed{f(1)=0}$

Thus, $x=1$ is the root of given polynomial.

when $x=-2$,

$f(-2)=2(-2)^3+(-2)^2-5(-2)+2$

$=-16+4+10+2$

$\boxed{f(-2)=0}$

Thus, $x=-2$ is the root of given polynomial.

Now, sum of zeros $=-b/a$

$1/2+1-1/2=-1/2$

$-1/2=-1/2$

and sum of products $=c/a$

$(1/2\times1)+(1)(-2)+(1/2)(-2)=-5/2$

$1/2-2-1=-5/2$

$-5/2=-5/2$

products of zeros $=-d/a$

$1/2\times1\times(-2)=-(2)/2$

$-1=-1$

Thus, the relationship betn the zeros and the coefficients is verified.

iii) $g(x) = x^3 - 4x^2 + 5x - 2$; 2, 1, 1

$\rightarrow$ The given polynomial is $g(x) = x^3 - 4x^2 + 5x - 2$

where $a = 1$, $b = -4$, $c = 5$, $d = -2$

when $x = 2$,

$g(2) = (2)^3 - 4(2)^2 + 5(2) - 2$

$= 8 - 16 + 10 - 2$

$\boxed{g(2) = 0}$

Thus, $x = 2$ is the zero of given polynomial.

when $x = 1$,

$g(1) = (1)^3 - 4(1)^2 + 5(1) - 2$

$= 1 - 4 + 5 - 2$

$\boxed{g(1) = 0}$

Thus, $x = 1$ is the zero of given polynomial.

Now, sum of zeros $= -b/a$

$1 + 1 + 2 = -(-4)/1$

$4 = 4$

Sum of products of the zeros taken two at a time $= c/a$

$(1 \times 1) + (1 \times 2) + (2 \times 1) = 5/1$

$1 + 2 + 2 = 5$

$5 = 5$

Now, products of zeros $= -d/a$

$1 \times 1 \times 2 = -(-2)/1$

$2 = 2$

Thus, the relationship betn the zeros and coefficients is verified.

Q.2.) Find a cubic polynomial with the sum, sum of the product of its zerose taken two at a time, and product of its zeros as 3, -1 and -3 respectively.

Soln→ We all know that,

the standard form of a cubic polynomial is

$$ax^3+bx^2+cx+d.$$

The cubic polynomial can be expressed in the form of roots is given as,

$$f(x) = k[x^3 - (\text{sum of roots})\,x^2 + (\text{sum of products of roots taken two at a time})\,x - (\text{product of roots})]$$

where, k is any non-zero real number.

Now, $f(x) = k[x^3 - 3x^2 + (-1)x - (-3)]$

$f(x) = k[x^3 - 3x^2 - x + 3]$

is the required cubic polynomial.

Q.3.) If the zeros of the polynomial $f(x) = 2x^3 - 15x^2 + 37x - 30$ are in A.P. find them.

Soln:- The given cubic polynomial is

$$f(x) = 2x^3 - 15x^2 + 37x - 30$$

where α, β & γ be the three roots of polynomial.

The given polynomial which has three roots and which are in A.P.

Now, we consider

$$\alpha = a-d,\quad \beta = a \text{ and } \gamma = a+d$$

where a is the first term & d is the common difference here.

But, from given polynomial,

$$a = 2,\ b = -15,\ c = 37\ \&\ d = 30$$

sum of roots = $\alpha+\beta+\gamma$

$= (a-d)+a+(a+d)$

$= 3a$

$= -b/a$

$= -(-15/2)$

sum of roots = $3a = 15/2$

$\Rightarrow \boxed{a = 5/2}$

Product of roots = $(a-d)\times(a)\times(a+d)$

$= a(a^2-d^2)$

$= -d/a$

$= -(30)/2 = 15$

Thus, $a(a^2-d^2) = 15$

by putting value of $a = 5/2$,

$5/2\left[(5/2)^2 - d^2\right] = 15$

$5(25/4 - d^2) = 30$

$25/4 - d^2 = 6$

$25 - 4d^2 = 24$

$1 = 4d^2$

$\therefore \boxed{d = 1/2 \text{ or } -1/2}$

when $d = 1/2$ & $a = 5/2 \Rightarrow$ the zeros are $2, 5/2$ & 3.

when $d = -1/2$ & $a = 5/2 \Rightarrow$ the zeros are $3, 5/2$ & 2.

Exercise 2.3

Q.1.) Apply division algorithm to find the quotient $q(x)$ and remainder $r(x)$ on dividing $f(x)$ by $g(x)$ in each of the following. i) $f(x) = x^3 - 6x^2 + 11x - 6$, $g(x) = x^2 + x + 1$

Sol^n:- Given that, $f(x) = x^3 - 6x^2 + 11x - 6$

and $g(x) = x^2 + x + 1$

$$\begin{array}{r|l} & x - 7 \\ \hline (x^2+x+1) & x^3 - 6x^2 + 11x - 6 \\ & \underline{\pm x^3 \pm x^2 \pm x} \\ & -7x^2 + 10x - 6 \\ & -7x^2 - 7x - 7 \\ & \underline{\quad + \quad + \quad +} \\ & 17x + 1 \end{array}$$

Thus, here

quotient $q(x) = x - 7$

Remainder $r(x) = 17x + 1$

ii) $f(x) = 10x^4 + 17x^3 - 62x^2 + 30x - 3$, $g(x) = 2x^2 + 7x + 1$

→ Given that, $f(x) = 10x^4 + 17x^3 - 62x^2 + 30x - 3$

And, $g(x) = 2x^2 + 7x + 1$

$$\begin{array}{r|l} & 5x^2 - 9x - 2 \\ \hline (2x^2+7x+1) & 10x^4 + 17x^3 - 62x^2 + 30x - 3 \\ & 10x^4 + 35x^3 + 5x^2 \\ & \underline{- \quad - \quad -} \\ & -18x^3 - 67x^2 + 30x - 3 \\ & -18x^3 - 63x^2 - 9x \\ & \underline{\quad + \quad + \quad +} \\ & -4x^2 + 39x - 3 \\ & -4x^2 - 14x - 2 \\ & \underline{\quad + \quad + \quad +} \\ & 53x - 1 \end{array}$$

Thus, quotient $q(x) = 5x^2 - 9x - 2$

Remainder $r(x) = 53x - 1$

iii) $f(x) = 4x^3 + 8x^2 + 8x + 7$, $g(x) = 2x^2 - x + 1$

$\rightarrow$ The given polynomial is $f(x) = 4x^3 + 8x^2 + 8x + 7$

and $g(x) = 2x^2 - x + 1$

$$
\begin{array}{r|l}
 & \quad 2x + 5 \\
\hline
(2x^2 - x + 1) & 4x^3 + 8x^2 + 8x + 7 \\
 & 4x^3 - 2x^2 + 2x \\
 & - \quad + \quad - \\
\hline
 & \quad 10x^2 + 6x + 7 \\
 & \quad 10x^2 - 5x + 5 \\
 & \quad - \quad + \quad - \\
\hline
 & \quad\quad 11x + 2
\end{array}
$$

Thus,

quotient $q(x) = 2x + 5$

Remainder $r(x) = 11x + 2$

iv) $f(x) = 15x^3 - 20x^2 + 13x - 12$, $g(x) = x^2 - 2x + 2$

$\rightarrow$ The given polynomial is $f(x) = 15x^3 - 20x^2 + 13x - 12$

and $g(x) = x^2 - 2x + 2$

$$
\begin{array}{r|l}
 & \quad 15x + 10 \\
\hline
x^2 - 2x + 2 & 15x^3 - 20x^2 + 13x - 12 \\
 & 15x^3 - 30x^2 + 30x \\
 & - \quad + \quad - \\
\hline
 & \quad 10x^2 - 17x - 12 \\
 & \quad 10x^2 - 20x + 20 \\
 & \quad - \quad + \quad - \\
\hline
 & \quad\quad 3x - 32
\end{array}
$$

Thus, quotient $q(x) = 15x + 10$

Remainder $r(x) = 3x - 32$

Q.2 Check whether the first polynomial is a factor of the second polynomial by applying the division algorithm.

Soln→ i) $g(t) = t^2 - 3$; $f(t) = 2t^4 + 3t^3 - 2t^2 - 9t - 12$

Given that, $g(t) = t^2 - 3$ and

$$f(t) = 2t^4 + 3t^3 - 2t^2 - 9t - 12$$

$$\begin{array}{r|l}
 & 2t^2 + 3t + 4 \\
t^2 - 3 & 2t^4 + 3t^3 - 2t^2 - 9t - 12 \\
 & \underline{2t^4 + 0t^3 - 6t^2} \\
 & \quad 3t^3 + 4t^2 - 9t - 12 \\
 & \quad \underline{3t^3 + 0t^2 - 9t} \\
 & \qquad 4t^2 + 0t - 12 \\
 & \qquad \underline{4t^2 + 0t - 12} \\
 & \qquad\quad 0
\end{array}$$

Thus,

quotient $q(t) = 2t^2 + 3t + 4$

Remainder

$r(t) = 0$

Here As, remainder $r(t) = 0$, so we can say that the first polynomial $g(t)$ is the factor of second polynomial $f(t)$.

ii) $g(x) = x^3 - 3x + 1$; $f(x) = x^5 - 4x^3 + x^2 + 3x + 1$

→ The given polynomial is $g(x) = x^3 - 3x + 1$, $f(x) = x^5 - 4x^3 + x^2 + 3x + 1$

Now,

$$\begin{array}{r|l}
 & x^2 \\
(x^3 - 3x + 1) & x^5 + 0x^4 - 4x^3 + x^2 + 3x + 1 \\
 & \underline{x^5 + 0x^4 - 3x^3 + x^2} \\
 & \qquad 0x^2 + 3x + 1 \\
 & \qquad\quad \underline{3x - 1} \\
 & \qquad\qquad 2
\end{array}$$

Thus, quotient $q(x) = x^2$

Remainder $r(x) = 2$

Here, As remainder $r(x) = 2$ is not equal to zero & hence we can say that first polynomial is not the factor of second polynomial.

iii) $g(x) = 2x^2 - x + 3$; $f(x) = 6x^5 - x^4 + 4x^3 - 5x^2 - x - 15$

→ The given polynomials are $g(x) = 2x^2 - x + 3$ and

$$f(x) = 6x^5 - x^4 + 4x^3 - 5x^2 - x - 15$$

$$
\begin{array}{r|l}
 & 3x^3 + x^2 - 2x - 5 \\
\hline
(2x^2 - x + 3) & 6x^5 - x^4 + 4x^3 - 5x^2 - x - 15 \\
 & 6x^5 - 3x^4 + 9x^3 \\
 & - \quad + \quad - \\
\hline
 & 2x^4 - 5x^3 - 5x^2 - x - 15 \\
 & 2x^4 - x^3 + 3x^2 \\
 & - \quad + \quad - \\
\hline
 & -4x^3 - 8x^2 - x - 15 \\
 & -4x^3 + 2x^2 - 6x \\
 & + \quad - \quad + \\
\hline
 & -10x^2 + 5x - 15 \\
 & -10x^2 + 5x - 15 \\
 & + \quad - \quad + \\
\hline
 & 0
\end{array}
$$

Here, quotient $q(x) = 3x^3 + x^2 - 2x - 5$

and remainder $r(x) = 0$

Thus, as $r(x) = 0$, we can say that the first polynomial is the factor of the second polynomial.

Q.3.) Obtain all zeroes of the polynomial $f(x) = 2x^4 + x^3 - 14x^2 - 19x - 6$; if two of its zeroes are -2 and -1.

Soln → The given polynomial is

$$f(x) = 2x^4 + x^3 - 14x^2 - 19x - 6$$

Also, given that -2 and -1 are two zeroes of the given polynomial $f(x)$.

$\Rightarrow$ $(x+2)$ & $(x+1)$ are the factors of $f(x)$.

$$\Rightarrow (x+2)(x+1) = x^2 + x + 2x + 2$$

$$= x^3 + 3x + 2 \quad \text{——— (1)}$$

So, we can say that equⁿ(1) is the factor of $f(x)$.

By division algorithm,

$$
\begin{array}{r|l}
 & 2x^2-5x-3 \\
\hline
x^2+3x+2 & 2x^4+x^3-14x^2-19x-6 \\
 & 2x^4+6x^3+4x^2 \\
 & - \quad - \quad - \\
\hline
 & -5x^3-18x^2-19x-6 \\
 & -5x^3-15x^2-10x \\
 & + \quad + \quad + \\
\hline
 & -3x^2-9x-6 \\
 & -3x^2-9x-6 \\
 & + \quad + \quad + \\
\hline
 & 0
\end{array}
$$

Here, the quotient $q(x) = 2x^2-5x-3$

$f(x) = (2x^2-5x-3)(x^2+3x+2)$

Now, we take $2x^2-5x-3=0$ to find other zeroes.

$\Rightarrow 2x^2-5x-3=0$

$(2x+1)(x-3)=0$

$\boxed{x=-1/2}$ or $\boxed{x=3}$

Thus, all the zeroes of given polynomial are:

$-2, -1, -1/2$ and 3.

Q.4) Obtain all the zeroes of $f(x) = x^3+13x^2+32x+20$; if one of its zeroes is -2.

Solⁿ→ The given polynomial is $f(x) = x^3+13x^2+32x+20$

Also, $x=-2$ is the root of given polynomial.

$\Rightarrow (x+2)$ is the factor of polynomial $f(x)$.

By division algorithm,

$$
\begin{array}{r|l}
 & x^2+11x+10 \\
\hline
(x+2) & x^3+13x^2+32x+20 \\
 & \underline{x^3+2x^2} \\
 & \quad -\ \ - \\
 & \quad 11x^2+32x+20 \\
 & \quad \underline{11x^2+22x} \\
 & \quad -\ \ - \\
 & \qquad 10x+20 \\
 & \qquad \underline{10x+20} \\
 & \qquad -\ \ - \\
 & \qquad\quad 0
\end{array}
$$

Here, quotient $q(x) = x^2+11x+10$

and remainder $r(x) = 0$

So, $f(x) = (x^2+11x+10)(x+2)$

To find remaining zeroes, we can take

$$x^2+11x+10=0$$

$$\Rightarrow (x+10)(x+1)=0$$

$$\boxed{x=-10} \text{ or } \boxed{x=-1}$$

Thus, the all zeros of given polynomial are : $-10, -2, -1$.

Q.5) obtain all zeros of the polynomial $f(x) = x^4-3x^3-x^2+9x-6$, if the two of its zeroes are $-\sqrt{3}$ and $+\sqrt{3}$.

Soln:- The given polynomial is $f(x) = x^4-3x^3-x^2+9x-6$

And also $x=-\sqrt{3}$ and $x=+\sqrt{3}$ are the two zeroes of given polynomial. $f(x)$.

$\Rightarrow$ $(x+\sqrt{3})$ and $(x-\sqrt{3})$ are factors of $f(x)$.

$\Rightarrow$ (x^2-3) is also a factor of $f(x)$

By division algorithm,

$$
\begin{array}{r|l}
 & x^2 - 3x + 2 \\
\hline
x^2 - 3 & x^4 - 3x^3 - x^2 + 9x - 6 \\
 & \underline{\begin{matrix} x^4 + 0x^3 - 3x^2 \\ - \quad - \quad + \end{matrix}} \\
 & -3x^3 + 2x^2 + 9x - 6 \\
 & \underline{\begin{matrix} -3x^3 + 0x^2 + 9x \\ + \quad - \quad - \end{matrix}} \\
 & 2x^2 + 0x - 6 \\
 & \underline{\begin{matrix} 2x^2 + 0x - 6 \\ - \quad - \quad + \end{matrix}} \\
 & 0
\end{array}
$$

So, $f(x) = (x^2 - 3x + 2)(x^2 - 3)$

To find remaining zeroes of polynomial $f(x)$,

we take, $(x^2 - 3x + 2) = 0$

$\Rightarrow (x-2)(x-1) = 0$

$\boxed{x = 2}$ or $\boxed{x = 1}$

Thus, all the zeroes of given polynomial $f(x)$ are:

$-\sqrt{3}, 1, \sqrt{3}$ and 2.

Q. 6.) Obtain all the zeroes of the polynomial $f(x) = 2x^4 - 2x^3 - 7x^2 + 3x + 6$, if the two of it's zeroes are $-\sqrt{3/2}$ and $\sqrt{(3/2)}$.

Soln:- The given polynomial is

$$f(x) = 2x^4 - 2x^3 - 7x^2 + 3x + 6$$

Also, given that $x = -\sqrt{(3/2)}$ and $x = +\sqrt{(3/2)}$ are two roots of given polynomial.

$\Rightarrow$ $(x + \sqrt{3/2})$ and $(x - \sqrt{3/2})$ are the factors of polynomial $f(x)$.

$\Rightarrow$ $(x^2 - 3/2)$ is the factor of poly. $f(x)$.

By division algorithm,

$$
\begin{array}{r|l}
 & 2x^2-2x-4 \\
x^2-3/2 & 2x^4-2x^3-7x^2+3x+6 \\
 & 2x^4+0x^3-3x^2 \\
\hline
 & -2x^3-4x^2+3x+6 \\
 & -2x^3+0x^2+3x \\
\hline
 & -4x^2+0x+6 \\
 & -4x^2+0x+6 \\
\hline
 & 0
\end{array}
$$

Here, quotient $q(x) = (2x^2-2x-4)$

and remainder $r(x) = 0$

Thus, $f(x) = (2x^2-2x-4)(x^2-3/2)$

$= 2(x^2-x-2)(x^2-3/2)$

To find other roots, we take $(x^2-x-2) = 0$

$\Rightarrow (x-2)(x+1) = 0$

$\Rightarrow \boxed{x=2}$ or $\boxed{x=-1}$

Thus, all the zeroes of polynomial are : $-\sqrt{(3/2)}$, -1, $\sqrt{3/2}$ & 2.

Q. 7.) Find all the zeroes of the polynomial $x^4+x^3-34x^2-4x+120$, if the two roots are 2 and -2.

Soln:- The given polynomial is $f(x) = x^4+x^3-34x^2-4x+120$

Also, given that $x=2$ and $x=-2$ are two zeroes.

$\Rightarrow (x-2)$ & $(x+2)$ are factors of $f(x)$

$\Rightarrow (x^2-4)$ is a factor of $f(x)$

By division algorithm,

$$
\begin{array}{r}
x^2+x-30 \\
x^2-4 \overline{\smash{\big)} x^4+x^3-34x^2-4x+120} \\
\underline{\overset{-}{x^4}\overset{-}{+0x^3}\overset{+}{-4x^2}} \\
x^3-30x^2-4x+120 \\
\underline{\overset{-}{x^3}\overset{-}{+0x^2}\overset{+}{-4x}} \\
-30x^2+0x+120 \\
\underline{\overset{+}{-30x^2}\overset{-}{+0x}\overset{-}{+120}} \\
0
\end{array}
$$

Here, quotient $q(x) = x^2+x-30$

and remainder $r(x) = 0$

So, $f(x) = (x^2+x-30)(x^2-4)$

To find remaining zeroes, we will take

$$x^2+x-30 = 0$$

$$\Rightarrow (x+6)(x-5) = 0$$

$$\boxed{x=-6} \text{ or } \boxed{x=-5}$$

Thus, the remaining zeroes of polynomial are: $5, -2, 2$ & -6.