Chapter 4: Triangles

Exercise 4.1

Q.1.) Fill in the blanks using the correct word given in brackets:

1) All circles are similar. (Congruent, similar)
2) All squares are similar. (similar, congruent)
3) All equilateral triangles are similar (isosceles, equilaterals)
4) Two triangles are similar, if their corresponding angles are equal. (proportional, equal)
5) Two triangles are similar, if their corresponding sides are proportional. (proportional, equal)
6) Two polygons of the same number of sides are similar, if a) equal their corresponding angles are and their corresponding sides are b) proportional. (equal, proportional)

Exercise 4.2

Q.1) In a ΔABC, D and E are points on the sides AB and AC respectively such that $DE \parallel BC$.

i) If $AD = 6$ cm, $DB = 9$ cm and $AE = 8$ cm, find AC

→ Here, given that, In a ΔABC

D and E are points on the sides AB and AC respectively such that $DE \parallel BC$.

And, $AD = 6$ cm, $DB = 9$ cm and $AE = 8$ cm

To find AC:

According to Thales Theorem,

$$\frac{AD}{BD} = \frac{AE}{CE} \quad \because DE \parallel BC$$

Let us consider, $CE = x$

So then, we can write,

$$\frac{6}{9} = \frac{8}{x}$$

$$6x = 72 \text{ cm}$$

$$x = 72/6 \text{ cm}$$

$$\boxed{x = 12 \text{cm}}$$

Thus, $AC = AE + CE = 12 + 8 = 20$ cm

Here $\boxed{AC = 20 \text{cm}}$

ii) If $\frac{AD}{DB} = \frac{3}{4}$ and $AC = 15$ cm, find AE.

→ Here, given that $\frac{AD}{DB} = \frac{3}{4}$ & $AC = 15$ cm

To find AE:

By Thales Theorem,

$$\frac{AD}{BD} = \frac{AE}{EC} \quad \because DE \parallel BC$$

Let us consider, $AE = x$ then we can say that

$$CE = 15 - x$$

$$\Rightarrow \quad \frac{3}{4} = \frac{x}{(15-x)}$$

$$3(15-x) = 4x$$

$$45 - 3x = 4x$$

$$45 = 7x$$

$$x = 45/7$$

$$x = 6.43\,cm$$

Thus, here $\boxed{AE = 6.43\,cm}$

iii) If $\frac{AD}{DB} = \frac{2}{3}$ and $AC = 18\,cm$, find AE.

$\rightarrow$ Here, given that $\frac{AD}{DB} = \frac{2}{3}$ and $AC = 18\,cm$

To find AE:

According to Thales Theorem,

$$\frac{AD}{BD} = \frac{AE}{CE} \quad \because DE \parallel BC$$

Let us consider, $AE = x$ and $CE = 18 - x$

Now, $2/3 = \frac{x}{(18-x)}$

$$3x = 2(18-x)$$

$$3x = 36 - 2x$$

$$5x = 36$$

$$x = 7.2\,cm$$

Thus, here $\boxed{AE = 7.2\,cm}$

iv) If AD = 4 cm, AE = 8 cm, DB = $x-4$ cm and EC = $3x-19$, find x.

→ Here, given that AD = 4 cm, AE = 8 cm, DB = $x-4$ cm and EC = $3x-19$.

To find x:

By Thales Theorem,

$$\frac{AD}{BD} = \frac{AE}{ACE} \quad \because DE \parallel BC$$

Then, we can write,

$$\frac{4}{(x-4)} = \frac{8}{(3x-19)}$$

$$4(3x-19) = 8(x-4)$$

$$12x - 76 = 8x - 32$$

$$12x - 8x = 76 - 32$$

$$4x = 44\text{cm}$$

$$\boxed{x = 11\text{cm}}$$

v) If AD = 8 cm, AB = 12 cm and AE = 12 cm, find CE.

→ Here, given that

AD = 8 cm, AB = 12 cm and AE = 12 cm

To find CE:

According to Thales Theorem,

$$\frac{AD}{DB} = \frac{AE}{CE} \quad \because DE \parallel BC$$

$$\frac{8}{4} = \frac{12}{CE}$$

$$8 \times CE = 4 \times 12$$

$$CE = 48/8$$

$$\boxed{CE = 6\text{cm}}$$

vi) If AD = 4 cm, DB = 4.5 cm and AE = 8 cm, find AC.

→ Here, given that

AD = 4 cm, DB = 4.5 cm and AE = 8 cm

To find AC:

According to Thales theorem,

$$\frac{AD}{DB} = \frac{AE}{CE} \quad \because DE \parallel BC$$

$$\frac{4}{4.5} = \frac{8}{AC}$$

$$AC = (4.5 \times 8)/4$$

$$\boxed{AC = 9\,cm}$$

vii) If AD = 2 cm, AB = 6 cm and AC = 9 cm, find AE.

→ Here, given that

AD = 2 cm, AB = 6 cm and AC = 9 cm

To find AE:

By Thales Theorem,

$$\frac{AD}{BD} = \frac{AE}{CE} \quad \because DE \parallel BC$$

$$\frac{2}{4} = \frac{x}{(9-x)}$$

$$4x = 18 - 2x$$

$$6x = 18$$

$$x = 3\,cm$$

Thus, $\boxed{AE = 3\,cm}$

viii) If $\frac{AD}{BD} = \frac{4}{5}$ and $EC = 2.5$ cm, find AE.

→ Here, given that $\frac{AD}{BD} = \frac{4}{5}$, $EC = 2.5$ cm

To find AE:

According to Thales theorem,

$$\frac{AD}{BD} = \frac{AE}{CE} \quad \because DE \parallel BC$$

Then $\frac{4}{5} = \frac{AE}{2.5}$

$$AE = 4 \times 2.55/5$$

Thus, $\boxed{AE = 2cm}$

ix) If $AD = x$ cm, $DB = x-2$ cm, $AE = x+2$ cm, and $EC = x-1$ cm, find the value of x.

→ Here, given that $AD = x$ cm, $DB = (x-2)$ cm, $AE = (x+2)$ cm, $EC = (x-1)$ cm.

To find x:

By Thales Theorem,

$$\frac{AD}{BD} = \frac{AE}{CE} \quad \because DE \parallel BC$$

$$\frac{x}{(x-2)} = \frac{(x+2)}{(x-1)}$$

$$x(x-1) = (x-2)(x+2)$$

$$x^2 - x - x^2 + 4 = 0$$

$$\boxed{x = 4}$$

x) If $AD = 8x-7$ cm, $DB = 5x-3$ cm, $AE = 4x-3$ cm, and $EC = (3x-1)$ cm, find the value of x.

→ Here, given that $AD = (8x-7)$cm, $DB = (5x-3)$cm
$AE = (4x-3)$ cm and $EC = (3x-1)$cm.

To find x:

According to Thales theorem,

$$\frac{AD}{BD} = \frac{AE}{CE} \quad \because DE \parallel BC$$

$$\frac{(8x-7)}{(5x-3)} = \frac{(4x-3)}{(3x-1)}$$

$$(8x-7)(3x-1) = (5x-3)(4x-3)$$

$$24x^2 - 29x + 7 = 20x^2 - 27x + 9$$

$$4x^2 - 2x - 2 = 0$$

$$2(2x^2 - x - 1) = 0$$

$$2x^2 - x - 1 = 0$$

$$2x^2 - 2x + x - 1 = 0$$

$$2x(x-1) + 1(x-1) = 0$$

$$(x-1)(2x+1) = 0$$

$$\Rightarrow x = 1 \text{ or } x = -1/2$$

But, we all know that, side of triangle can never be negative.

Hence, we will take only positive value.

$$\therefore \boxed{x = 1}$$

xi) If $AD=(4x-3)$, $AE=(8x-7)$, $BD=(3x-1)$ and $CE=5x-3$, find the value of x.

$\longrightarrow$ Here, given that

$AD=(4x-3)$, $AE=(8x-7)$, $BD=(3x-1)$,

$EC=(5x-3)$

To find x:

According to Thales theorem,

$$\frac{AD}{BD}=\frac{AE}{CE} \quad \because DE \parallel BC$$

$$\frac{(4x-3)}{(3x-1)}=\frac{(8x-7)}{(5x-3)}$$

$$(4x-3)(5x-3)=(8x-7)(3x-1)$$

$$4x(5x-3)-3(5x-3)=3x(8x-7)-1(8x-7)$$

$$20x^2-12x-15x+9=24x^2-29x+7$$

$$20x^2-27x+9=24^2-29x+7$$

$$-4x^2+2x+2=0$$

$$4x^2-2x-2=0$$

$$4x^2-4x+2x-2=0$$

$$4x(x-1)+2(x-1)=0$$

$$(4x+2)(x-1)=0$$

$$\Rightarrow x=1 \text{ or } x=-2/4$$

We all know that, the side of a triangle can never be negative.

Hence we will take positive value only.

$$\therefore \boxed{x=1}$$

xii) If $AD = 2.5cm$, $BD = 3cm$, and $AE = 3.75cm$, find the length of AC.

→ Here, given that $AD = 2.5cm$, $BD = 3cm$, $AE = 3.75cm$

To find AC:

According to Thales theorem,

$$\frac{AD}{BD} = \frac{AE}{CE} \quad \because DE \parallel BC$$

$$\frac{2.5}{3} = \frac{3.75}{CE}$$

$$2.5 \times CE = 3.75 \times 3$$

$$CE = 3.75 \times 3 2.5$$

$$CE = 11.25$$

$$\boxed{CE = 4.5}$$

Thus, $AC = 3.75 + 4.5$

$$\therefore \boxed{AC = 8.25 \text{ cm}}$$

Q.2) In a $\triangle ABC$, D and E are points on the sides AB and AC respectively. For each of the following cases show that $DE \parallel BC$:

i) $AB = 12cm$, $AD = 8cm$, $AE = 12cm$ and $AC = 18cm$

→ To prove: $DE \parallel BC$

we have,

$AB = 12cm$, $AD = 8cm$, $AE = 12cm$, $AC = 18cm$

So, $BD = AB - AD = 12 - 8 = 4cm$

and $CE = AC - AE = 18 - 12 = 6cm$

It is seen that,

$$\frac{AD}{BD} = \frac{8}{4} = \frac{1}{2}$$

$$\frac{AE}{CE} = \frac{12}{6} = \frac{1}{2}$$

Thus, $\frac{AD}{BD} = \frac{AE}{CE}$

According to Thale's Theorem,

$\boxed{DE \parallel BC}$ Hence proved.

ii) $AB = 5.6\,cm$, $AD = 1.4\,cm$, $AC = 7.2\,cm$ and $AE = 1.8\,cm$

→ To prove $DE \parallel BC$:

Here, given that $AB = 5.6\,cm$, $AD = 1.4\,cm$, $AC = 7.2\,cm$

$AE = 1.8\,cm$

Now, $BD = AB - AD = 5.6 - 1.4 = 4.2\,cm$

and $CE = AC - AE = 7.2 - 1.8 = 5.4\,cm$

Now, $\frac{AD}{BD} = \frac{1.4}{4.2} = \frac{1}{3}$

$\frac{AE}{CE} = \frac{1.8}{5.4} = \frac{1}{3}$

Thus, $\frac{AD}{BD} = \frac{AE}{CE}$

According to converse of Thales theorem,

$\boxed{DE \parallel BC}$ Hence proved.

iii) $AB = 10.8\,cm$, $BD = 4.5\,cm$, $AC = 48\,cm$ and $AE = 2.8\,cm$

→ To prove $DE \parallel BC$:

given that, $AB = 10.8\,cm$, $BD = 4.5\,cm$, $AC = 4.8\,cm$

and $AE = 2.8\,cm$

Now, $AD = AB - DB = 10.8 - 4.5 = 6.3$

and $CE = AC - AE = 4.8 - 2.8 = 2$

So, $\frac{AD}{BD} = \frac{6.3}{4.5} = \frac{2.8}{2.0} = \frac{AE}{CE} = \frac{7}{5}$

According to Converse of Thales theorem,

$\boxed{DE \parallel BC}$ Hence proved.

iv) AD = 5.7 cm , BD = 9.5 cm , AE = 3.3 cm and EC = 5.5 cm

→ To prove DE || BC:

AD = 5.7 cm , BD = 9.5 cm, AE = 3.3 cm and EC = 5.5 cm

Now, $\frac{AD}{BD} = \frac{5.7}{9.5} = \frac{3}{5}$

and $\frac{AE}{CE} = \frac{3.3}{5.5} = \frac{3}{5}$

Thus, $\frac{AD}{BD} = \frac{AE}{CE}$

According to Thales Theorem Converse,

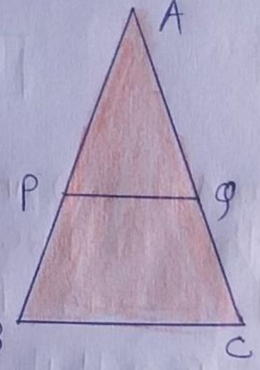
$\boxed{DE \parallel BC}$ Hence proved.

Q.3.) In a ΔABC, P and Q are the points on sides AB and AC respectively, such that PQ || BC , If AP = 2.4 cm , AQ = 2 cm, QC = 3 cm, ~~QE~~ and BC = 6 cm . Find AB and PQ.

Soln→ Given that, ΔABC , AP = 2.4 cm , AQ = 2 cm, QC = 3 cm and BC = 6 cm.

Also, PQ || BC

To find AB and PQ:



According to Thales theorem,

since , PQ || BC

$\frac{AP}{PB} = \frac{AQ}{QC}$

$\frac{2.4}{PB} = \frac{2}{3}$

$2 \times PB = 2.4 \times 3$

$PB = (2.4 \times 3)/2$

$\Rightarrow \boxed{PB = 3.6 \text{ cm}}$

Now, to find: $AB = AP + PB$

$AB = 2.4 + 3.6$

$\boxed{AB = 6 cm}$

Here, we consider ΔAPQ and ΔABC,

where, $\angle A = \angle A$

and $\angle APQ = \angle ABC$

Since, corresponding angles are equal, $PQ \parallel BC$ and AB is a transversal.

$\Rightarrow$ ΔAPQ and ΔABC are similar triangles ($\because$ by AA)

But, all we know that,

Corresponding parts of similar triangles are proportional,

$$\frac{AP}{AB} = \frac{PQ}{BC}$$

$\Rightarrow PQ = (AP/AB) \times BC$

$= (2.4/6) \times 6$

$\boxed{PQ = 2.4 cm}$

Q.4.) In a ΔABC, D and E are points on AB and AC respectively, such that $DE \parallel BC$. If $AD = 2.4 cm$, $AE = 3.2 cm$, $DE = 2 cm$ and $BC = 5 cm$. Find BD and CE.

$\rightarrow$ Here, Given that

In ΔABC such that $AD = 2.4 cm$, $AE = 3.2 cm$, $DE = 2 cm$ and $BC = 5 cm$.

Also, given that $BC \parallel DE$

To find BD and CE:

Since, $DE \parallel BC$, AB is traversal,

$\Rightarrow \angle APQ = \angle ABC$ $\because$ corresponding angles

Also, $DE \parallel BC$, AC is traversal

then $\Rightarrow \angle AED = \angle ACB$ $\because$ corresponding angles.

Now, In ΔADE and ΔABC,

$\angle ADE = \angle ABC$

$\angle AED = \angle ACB$

$\therefore \boxed{\Delta ADE = \Delta ABC}$ $\because$ by AA similarity condn

Now, we have

Corresponding parts of similar triangles are proportional.

$$\Rightarrow \frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$$

$$\Rightarrow \frac{AD}{AB} = \frac{DE}{BC}$$

$$\frac{2.4}{(2.4+DB)} = \frac{2}{5} \quad \because AB = AD + DB$$

$$2.4 + DB = 6$$

$$DB = 6 - 2.4$$

$$\boxed{DB = 3.6\text{ cm}}$$

Similarly, $AE/AC = DE/BC$

$3.2/(3.2+EC) = 2/5$ $\because AC = AE + EC$

$$3.2 + EC = 8$$

$$EC = 8 - 3.2$$

$$EC = 4.8\text{ cm}$$

$\therefore \boxed{BD = 3.6\text{ cm}}$ and $\boxed{CE = 4.8\text{ cm}}$

Exercise 4.3

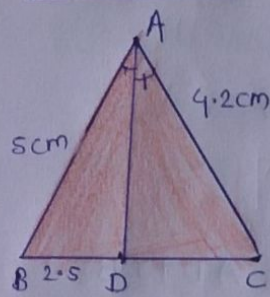
Q.1) In a $\triangle ABC$, AD is the bisector of $\angle A$, meeting side BC at D.

i) If $BD = 2.5cm$, $AB = 5cm$, and $AC = 4.2cm$ find DC.

→ Here, given that

In $\triangle ABC$, AD is the bisector of $\angle A$, meeting side BC at D. And $BD = 2.5cm$, $AB = 5cm$ and $AC = 4.2cm$.

To find DC:



Here, AD is the bisector of $\angle A$ meeting side BC at point D as shown in fig.

$\Rightarrow \frac{AB}{AC} = \frac{BD}{DC}$

$\frac{5}{4.2} = \frac{2.5}{DC}$

$5DC = 2.5 \times 4.2$

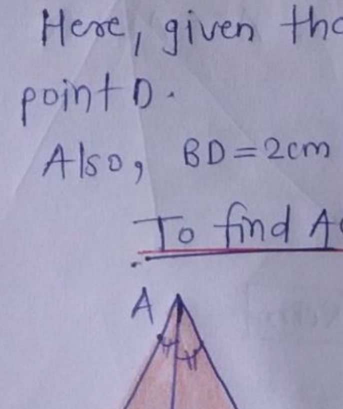
$\therefore \boxed{DC = 2.1 cm}$

ii) If $BD = 2cm$, $AB = 5cm$ and $DC = 3cm$, find AC.

→ Here, given that AD bisects $\angle A$, meeting side BC at point D.

Also, $BD = 2cm$, $AB = 5cm$ and $DC = 3cm$.

To find AC:



Here, AD is the bisector of angle A which meets at point D along BC.

$\Rightarrow \frac{AB}{AC} = \frac{BD}{DC}$

$\frac{5}{AC} = \frac{\cancel{5}\,2}{3}$

$2AC = 5 \times 3$

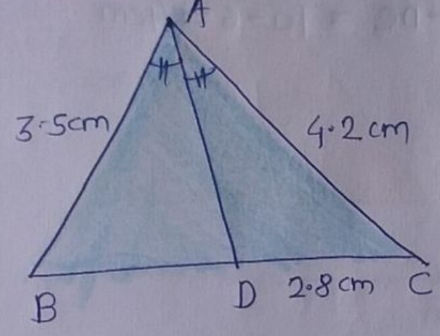
$\therefore \boxed{AC = 7.5cm}$

iii) If $AB = 3.5cm$, $AC = 4.2cm$ and $DC = 2.8cm$ find BD.

→ Here, In ΔABC, AD bisects angle A and which meets at point D along BC.

Also, $AB = 3.5cm$, $AC = 4.2cm$ and $DC = 2.8cm$

To find BD:



Here, in ΔABC, AD is the bisector of $\angle A$ and meets at point D along BC.

$$\Rightarrow \frac{AB}{AC} = \frac{BD}{DC}$$

$$\frac{3.5}{4.2} = \frac{BD}{2.8}$$

$$4.2 \times BD = 3.5 \times 2.8$$

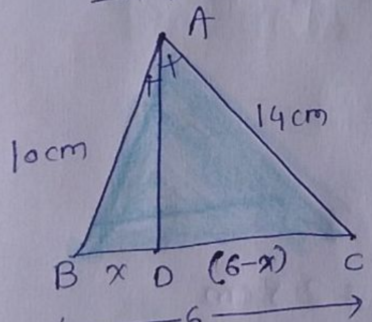
$$BD = 7/3$$

$$\therefore \boxed{BD = 2.3 cm}$$

iv) If $AB = 10cm$, $AC = 14cm$, and $BC = 6cm$, find BD and DC.

→ Here, given that In ΔABC, AD is the bisector of $\angle A$ meeting side BC at point D.

And $AB = 10cm$, $AC = 14cm$, $BC = 6cm$



To find BD and DC:

Here, AD is the angle bisector of A and it meets at point D along BC.

$$\Rightarrow AB/AC = BD/DC$$

$$\frac{10}{14} = \frac{x}{(6-x)}$$

$$14x = 60 - 6x$$

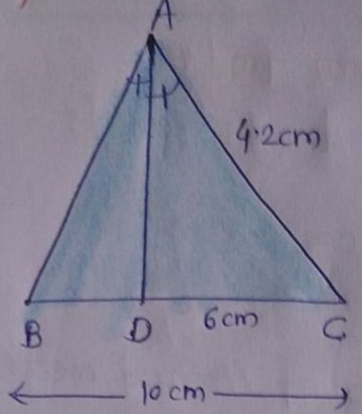
$$20x = 60$$

$$x = 3 \quad \text{and } DC = 6 - 3$$

$$\boxed{BD = 3} \qquad \boxed{DC = 3cm}$$

v) If $AC = 4.2$ cm, $DC = 6$ cm, and $BC = 10$ cm, find AB

→



Here, In ΔABC,

AD bisects angle A, which meets at point D along BC.

$$\Rightarrow \frac{AB}{AC} = \frac{BD}{DC}$$

$$\frac{AB}{4.2} = \frac{BD}{6}$$

Here, $BD = BC - DC = 10 - 6 = 4$ cm

$$\Rightarrow \frac{AB}{4.2} = \frac{4}{6}$$

$$AB = (2 \times 4.2)/\cancel{6}3$$

$\therefore \boxed{AB = 2.8 \text{ cm}}$

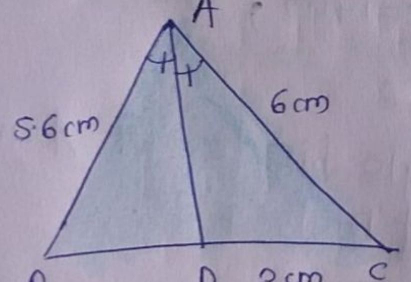
vi) If $AB = 5.6$ cm, $AC = 6$ cm, and $DC = 3$ cm, find BC.

→ Here, given that,

AD bisects angle A and meets at point D along BC.

Also, $AB = 5.6$ cm, $AC = 6$ cm and $DC = 3$ cm

To find BC:



Here, as AD is angle bisector,

$$\frac{AB}{AC} = \frac{BD}{DC}$$

$$\frac{5.6}{6} = \frac{BD}{3}$$

$$BD = 5.6/2 = 2.8 \text{ cm}$$

we have,

$$BD = BC - DC$$

$$2.8 = BC - 3$$

$\therefore \boxed{BC = 5.8 \text{ cm}}$

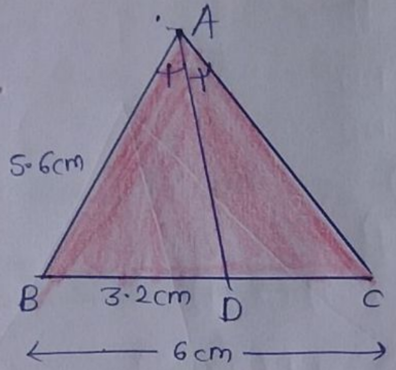
vii) If AB = 5.6 cm, BC = 6 cm and BD = 3.2 cm find AC.

→ Here, In ΔABC,

AD is the angle bisector of $\angle A$ and meets at point D along BC.

Also, given that AB = 5.6 cm, BC = 6 cm, BD = 3.2 cm

To find AC:



Here, as AD is angle bisector,

$$\frac{AB}{AC} = \frac{BD}{DC}$$

$$\frac{5.6}{AC} = \frac{3.2}{DC}$$

we have,

$$BD = BC - DC$$

$$3.2 = 6 - DC$$

$$\therefore \boxed{DC = 2.8 cm}$$

$$\Rightarrow \frac{5.6}{AC} = \frac{3.2}{2.8}$$

$$AC = (5.6 \times 2.8)/3.2$$

$$\boxed{AC = 4.9 cm}$$

viii) If AB = 10 cm, AC = 6 cm and BC = 12 cm, find BD and DC.

→ Here, In ΔABC

AD is the angle bisector of angle A which meets at point D along BC.

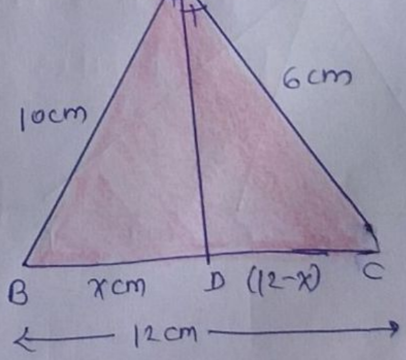
Also, AB = 10 cm, AC = 6 cm, BC = 12 cm

To find DC:

As AD is the bisector of $\angle A$,

$$\Rightarrow \frac{AB}{AC} = \frac{BD}{DC}$$

$$\frac{10}{6} = \frac{BD}{DC}$$



we have,

$$BD = BC - DC$$
$$= 12 - DC$$

Let us suppose, $BD = x$

$$DC = 12 - x$$

Now, $\frac{10}{6} = \frac{BD}{DC}$

$$\frac{10}{6} = \frac{x}{(12-x)}$$
$$5(12-x) = 3x$$
$$60 - 5x = 3x$$
$$\therefore x = \frac{60}{8} = 7.5$$

Thus, $DC = 12 - 7.5 = 4.5$ cm

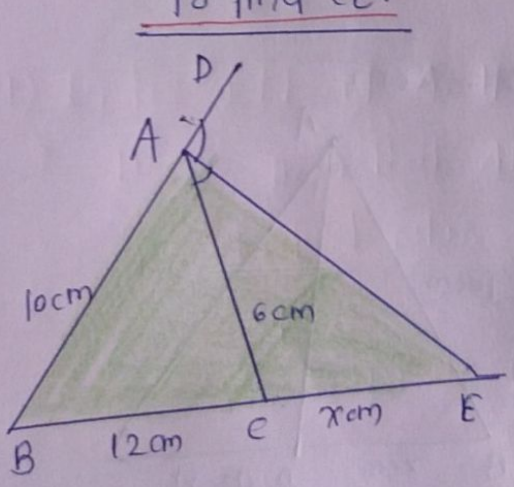
$\boxed{DC = 4.5cm}$ and $\boxed{BD = 7.5cm}$

Q.2.) In figure 4.57, AE is the bisector of the exterior $\angle CAD$ meeting BC produced in E. If $AB = 10cm$, $AC = 6$ cm and $BC = 12cm$, find CE.

→ Here, given that,

AE is the bisector of the exterior $\angle CAD$ and $AB = 10$ cm, $AC = 6$ cm and $BC = 12$ cm

To find CE:



Here, in $\triangle ABC$, AE is angle bisector of exterior $\angle CAD$.

$$\Rightarrow \frac{BE}{CE} = \frac{AB}{AC}$$

Let us consider, $CE = x$

$$\Rightarrow \frac{BE}{CE} = \frac{AB}{AC}$$

$$\frac{(12+x)}{x} = \frac{10}{6}$$

$$6x + 72 = 10x$$
$$10x - 6x = 72$$
$$4x = 72$$
$$\therefore x = 18$$

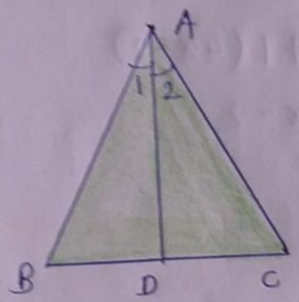
Thus, $\boxed{CE = 18\text{ cm}}$

Q.3) In fig. 4.58, ΔABC is a triangle such that $\frac{AB}{AC} = \frac{BD}{DC}$, $\angle B = 70°$, $\angle C = 50°$ find $\angle BAD$.

Sol^n →

Here, In ΔABC, $\frac{AB}{AC} = \frac{BD}{DC}$, $\angle B = 70°$ and $\angle C = 50°$

To find $\angle BAD$:



we have,

In ΔABC, $\angle A = 180 - (70 + 50)$

$= 180 - 120$

$\boxed{\angle A = 60°}$

Also, $\frac{AB}{AC} = \frac{BD}{DC}$

⇒ AD is the angle bisector of angle A.

Thus, we can write,

$$\angle BAD = \angle A/2 = 60/2 = 30°$$

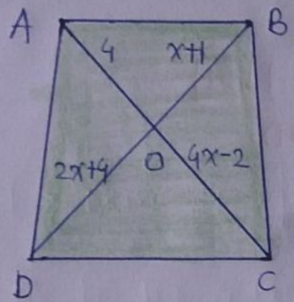
$$\boxed{\angle BAD = 30°}$$

Exercise: 4.4

Q.1.) i) In fig. 4.70, if AB ∥ CD, find the value of x.

⟶ Here, given that AB ∥ CD.

To find x:



Here, diagonals of a parallelogram bisects each other.

Thus, $\frac{AO}{CO} = \frac{BO}{DO}$

$$\frac{4}{(4x-2)} = \frac{(x+1)}{(2x+4)}$$

$$4(2x+4) = (4x-2)(x+1)$$

$$8x+16 = x(4x-2)+1(4x-2)$$

$$8x+16 = 4x^2-2x+4x-2$$

$$-4x^2+8x+16+2-2x=0$$

$$-4x^2+6x+8=0$$

$$4x^2-6x-18=0$$

$$4x^2-12x+6x-18=0$$

$$4x(x-3)+6(x-3)=0$$

$$(4x+6)(x-3)=0$$

$$\Rightarrow \boxed{x=-6/4} \text{ or } \boxed{x=3}$$

ii) In fig. 4.71, if AB ∥ CD, find the value of x.

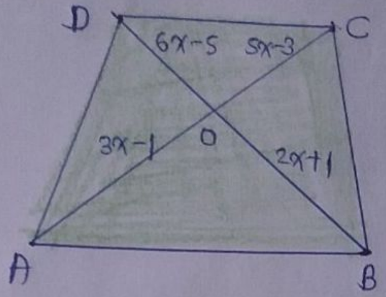
⟶ Here, given that AB ∥ CD

To find value of x:

we have,

Diagonals of a parallelogram bisects each other.

Then, $\frac{AO}{CO} = \frac{BO}{DO}$



$\frac{(6x-5)}{(2x+1)} = \frac{(5x-3)}{(3x-1)}$

$(6x-5)(3x-1) = (2x+1)(5x-3)$

$3x(6x-5) - 1(6x-5) = 2x(5x-3) + 1(5x-3)$

$\Rightarrow 18x^2 - 10x^2 - 21x + 5 + x + 3 = 0$

$8x^2 - 16x - 4x + 8 = 0$

$8x(x-2) - 4(x-2) = 0$

$(8x-4)(x-2) = 0$

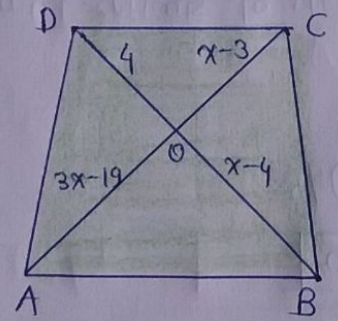
$x = 4/8 = 1/2$ or $x = -2$

$\therefore \boxed{x = 1/2}$

iii) In fig. 4.72, if AB || CD. If OA = 3x-19, OB = x-4, OC = x-3 and OD = 4, find x.

→ Here, given that AB || CD.

To find value of x:



we have,
the diagonals of a parallelogram bisects each other
Then, we can write,

$\frac{AO}{CO} = \frac{BO}{DO}$

$\frac{(3x-19)}{(x-3)} = \frac{(x-4)}{4}$

$4(3x-19) = (x-3)(x-4)$

$12x - 76 = x(x-4) - 3(x-4)$

$12x - 76 = x^2 - 4x - 3x + 12$

$\therefore -x^2 + 7x - 12 + 12x - 76 = 0$

$$-x^2+19x-88=0$$
$$x^2-19x+88=0$$
$$x^2-11x-8x+88=0$$
$$x(x-11)-8(x-11)=0$$
$$\boxed{x=11} \text{ or } \boxed{x=8}$$

Exercise 4.5

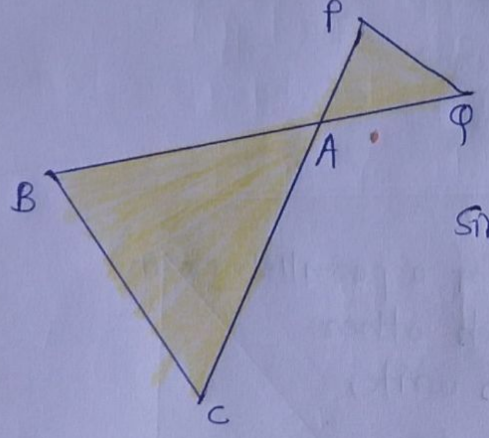
Q.1) In fig. 4.136, $\Delta ACB \sim \Delta APQ$. If $BC = 8$ cm, $PQ = 4$ cm, $BA = 6.5$ cm and $AP = 2.8$ cm. find CA and AQ.

Soln → Here, given that

$$\Delta ACB \sim \Delta APQ$$

$$BC = 8\text{cm},\ PQ = 4\text{cm},\ BA = 6.5\text{cm},\ AP = 2.8\text{cm}$$

To find CA and AQ:



we have,

$$\Delta ACB \sim \Delta APQ$$

$$\Rightarrow \frac{BA}{AQ} = \frac{CA}{AP} = \frac{BC}{PQ}$$

Since, corresponding parts of similar triangle.

$$\Rightarrow \frac{6.5}{AQ} = \frac{8}{4}$$

$$AQ = (6.5 \times 4)/8$$

$$\boxed{AQ = 3.25\text{cm}.}$$

In similar manner,

$$\frac{CA}{AP} = \frac{BC}{PQ}$$

$$CA/2.8 = 8/4$$

$$CA = 2.8 \times 2$$

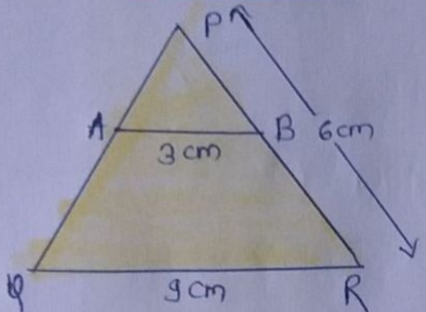
$$\boxed{CA = 5.6\text{cm}}$$

Q.2.) In fig. 4.137, AB || QR, find the length of PB.

→ Here, given that AB || QR

and AB = 3 cm, QR = 9 cm and PR = 6 cm

To find PB:



In ΔPAB and ΔPQR,

we have,

$\angle P = \angle P$ ∵ common angle

$\angle PAB = \angle PQR$ ∵ corresponding angles

$\Rightarrow \boxed{\Delta PAB \sim \Delta PQR}$ ∵ by AA

Hence, $\frac{AB}{QR} = \frac{PB}{PR}$ ∵ corresponding parts of similar triangles are proportional

$\Rightarrow \frac{3}{9} = \frac{PB}{6}$

$\Rightarrow PB = 6/3$

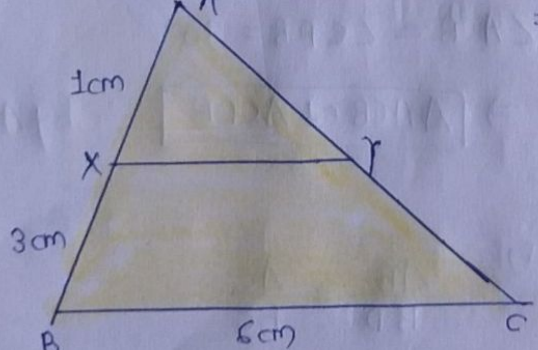
$\boxed{PB = 2\text{ cm}}$

Q.3.) In fig. 4.138 given, XY || BC, find the length of XY.

Soln.– Here, given that XY || BC

Also, AX = 1cm, XB = 3cm and BC = 6 cm

To find XY:



In ΔAXY & ΔABC

$\Rightarrow \angle A = \angle A$ ∵ common angle

$\angle AXY = \angle ABC$

$\Rightarrow \boxed{\Delta AXY \sim \Delta ABC}$

Thus, corresponding parts of similar triangles are proportional.

$$\frac{XY}{BC} = \frac{AX}{AB}$$

we have, $AB = AX + XB = 1 + 3 = 4$

$$\Rightarrow \frac{XY}{6} = \frac{1}{4}$$

$$XY = 6/4$$

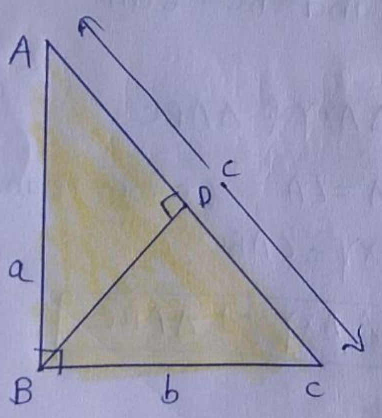
Thus, $\boxed{XY = 1.5\text{ cm}}$

Q.4.) In a right-angled triangle with sides a and b and hypotenuse c, the altitude drawn on the hypotenuse is x. Prove that $ab = cx$.

Solⁿ → Now, we consider ΔABC as a right-angled triangle which is having sides a and b, and c as a hypotenuse.

Let we have drawn an altitude BD on AC.

To prove $ab = cx$:



we have,

In ΔACB and ΔCDB here,

$\angle B = \angle B$ ∵ common angle

$\angle ACB = \angle CDB = 90°$

$\Rightarrow \boxed{\Delta ACB \sim \Delta CDB}$ ∵ by AA

Thus,

$$\frac{AB}{BD} = \frac{AC}{BC}$$

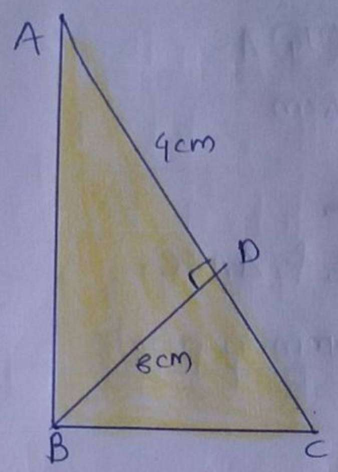
$$\frac{a}{x} = \frac{c}{b}$$

$$\Rightarrow \boxed{xc = ab}$$

Hence proved.

Q.5) In fig. 4.139, $\angle ABC = 90$ and $BD \perp^{er} AC$. If $BD = 8cm$, and $AD = 4cm$. find CD.

Soln →



Here, given that

$\angle ABC = 90°$ and $BD \perp^{er} AC$

$BD = 8$ cm

$AD = 4$cm

To find CD:

we have,

In ΔABC, $BD \perp^{er} AC$

$\Rightarrow \Delta DBA \sim \Delta DCB$ ∵ by AA

Then, we can write

$$\frac{BD}{CD} = \frac{AD}{BD}$$

$$BD^2 = AD \times DC$$

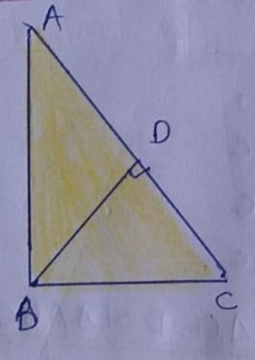
$$(8)^2 = 4 \times DC$$

$$DC = 64/4 = 16cm$$

Thus, $\boxed{CD = 16cm}$

Q.6) In fig. 4.140, $\angle ABC = 90°$ and $BD \perp^{er} AC$. If $AC = 5.7cm$, $BD = 3.8$ cm and $CD = 5.4$ cm, find BC.

Soln:—



Here, given that

$\angle ABC = 90°$, $BD \perp^{er} AC$

and $AC = 5.7cm$, $BD = 3.8cm$, $CD = 5.4cm$

To find BC:

we have, $\Delta ABC \sim \Delta BDC$ ∵ by AA

$\Rightarrow \angle BCA = \angle DCA = 90°$

$\angle AB\cancel{C} = \angle ABC$ ∵ common

Thus, we can write

$$\frac{AB}{BD} = \frac{BC}{CD}$$

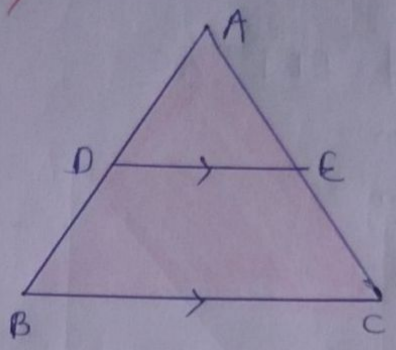
$\boxed{BC = 8.1cm}$

$$\frac{5.7}{3.8} = \frac{BC}{5.4}$$

$$BC = (5.7 \times 5.4)/3.8 = 8.1$$

Q.7) In the fig. 4.141 given, DE || BC such that, $AE = (1/4) AC$. If $AB = 6$ cm, find AD.

→



Here, given that $DE \parallel BC$

and $AE = (1/4) AC$

$AB = 6$ cm

To find AD:

Here, In ΔADE & ΔABC,

$\angle A = \angle A$ ∵ common angle

$\angle ADE = \angle ABC$

$\Rightarrow \boxed{\Delta ADE \sim \Delta ABC}$

Thus, $\frac{AD}{AB} = \frac{AE}{AC}$ ∵ corresponding parts of similar triangles are proportional

$\Rightarrow \frac{AD}{6} = 1/4$

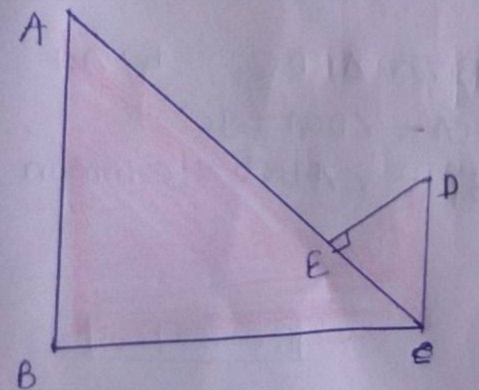
$4 \times AD = 6$

$AD = 6/4$

$\Rightarrow \boxed{AD = 1.5 \text{ cm}}$

Q.8) In the fig. 4.142 given, If $AB \perp BC$, and $DE \perp AC$, prove that $\Delta CED \sim \Delta ABC$.

Soln:-



Here, given that,

$AB \perp^{er} BC$

$DC \perp^{er} BC$

$DE \perp^{er} AC$

To prove: $\Delta CED \sim \Delta ABC$

we have,

from ΔABC & ΔCED,

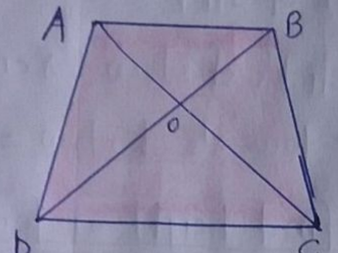
$\angle B = \angle E = 90^{\circ}$

$\angle BAC = \angle ECD$

$\Rightarrow \boxed{\Delta CED \sim \Delta ABC}$ ∵ by AA

Q.9.) Diagonals AC and BD of a trapezium ABCD with AB ∥ DC intersect each other at the point O. Using similarity criterion for two triangles, show that OA/OC = OB/OD

Soln:— Here, given that OC is the point of intersection of AC and BD in the trapezium ABCD. where AB ∥ DC.



To prove $\frac{OA}{OC} = \frac{OB}{OD}$:

we have,

In ΔAOB and ΔCOD

$\Rightarrow \angle AOB = \angle COD$ ∵ opposite angles

$\angle OAB = \angle OCD$

Then, $\Delta AOB \sim \Delta COD$

Thus, $\boxed{\frac{OA}{OC} = \frac{OB}{OD}}$

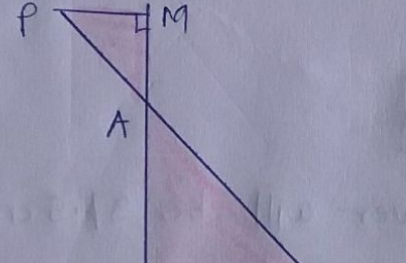
Hence proved.

Q.10.) If ΔABC and ΔAMP are two right triangles, right angled at B and M, respectively such that $\angle MAP = \angle BAC$.

Prove that: i) $\Delta ABC \sim \Delta AMP$

ii) CA/PA = BC/MP

Soln:— i) Here, given that, ΔABC & ΔAMP are the two right angled triangles.



we have,

$\angle AMP = \angle B = 90°$

$\angle MAP = \angle BAC$

$\Rightarrow \boxed{\Delta ABC \sim \Delta AMP}$ ∵ by AA

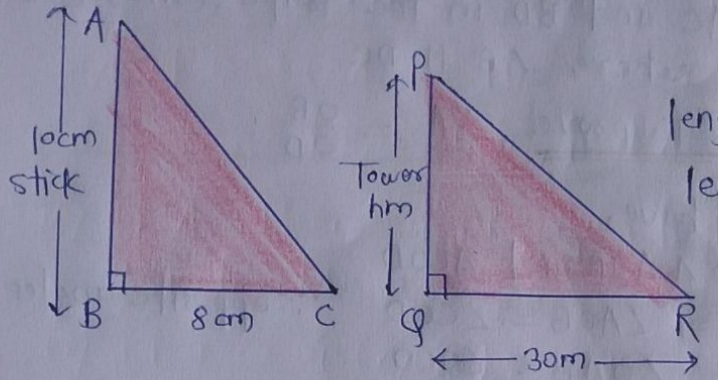
ii) As $\Delta ABC \sim \Delta AMP$

$\Rightarrow \frac{CA}{PA} = \frac{BC}{MP}$

Hence proved.

Q.11) A vertical stick 10 cm long casts a shadow 8 cm long. At the same time, a tower casts a shadow 30 m long. Determine the height of the tower.

Soln:—



Here, given that

length of stick = 10 cm

length of stick's shadow = 8 cm

length of tower's shadow = 3000 cm

To find height of tower:

In $\Delta ABC \sim \Delta PQR$,

$\angle ABC = \angle PQR = 90°$

$\angle ACB = \angle PRQ$

$\Rightarrow \boxed{\Delta ABC \sim \Delta PQR}$ $\because$ By AA

Thus, we can write,

$$\frac{AB}{BC} = \frac{PQ}{QR}$$

$$\frac{10}{8} = \frac{PQ}{3000}$$

$$PQ = (3000 \times 10)/8$$

$$PQ = 30000/8$$

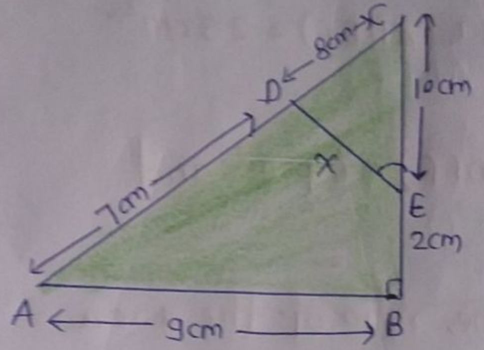
$$PQ = 3750/100$$

Thus, $\boxed{PQ = 37.5 m}$

Thus, the height of the tower will be 37.5 cm.

Q.12) In fig. 4.143, $\angle A = \angle CED$, prove that $\Delta CAB \sim \Delta CED$.
Also find the value of x.

Sol^n:—



Here, given that,

$\angle A = \angle CED$

To prove: $\Delta CAB \sim \Delta CED$

In $\Delta CAB \sim \Delta CED$,

$\angle C = \angle C$, ∵ common

$\angle A = \angle CED$

$\Rightarrow \boxed{\Delta CAB \sim \Delta CED}$ ∵ by AA

Hence, we can write as,

$$\frac{CA}{CE} = \frac{AB}{ED}$$

$$\frac{15}{10} = 9/x$$

$$x = (9 \times 10)/15$$

$$\boxed{x = 6cm}$$

Exercise 4.6

Q.1) Triangles ABC and DEF are similar.

i) If area of $(\triangle ABC) = 16\,cm^2$, area $(\triangle DEF) = 25\,cm^2$ and $BC = 2.3\,cm$, find EF.

ii) If area $(\triangle ABC) = 9\,cm^2$, area $(\triangle DEF) = 64\,cm^2$ and $DE = 5.1\,cm$, find AB.

iii) If $AC = 19\,cm$ and $DF = 8\,cm$, find the ratio of the area of two triangles.

iv) If area of $(\triangle ABC) = 36\,cm^2$, area $(\triangle DEF) = 64\,cm^2$ and $DE = 6.2\,cm$, find AB.

v) If $AB = 1.2\,cm$ and $DE = 1.4\,cm$, find the ratio of the area of two triangles.

Soln:- As we allready know that,
The ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides,

i) $$\frac{\text{area}(\triangle ABC)}{\text{area}(\triangle DEF)} = \left(\frac{BC}{EF}\right)^2 = \frac{16}{25}$$

$$\left(\frac{2.3}{EF}\right)^2 = \frac{16}{25}$$

$$\Rightarrow \frac{2.3}{EF} = \frac{4}{5}$$

$$\Rightarrow EF = \frac{5 \times 2.3}{4}$$

$$\boxed{EF = 2.875\,cm}$$

ii) $$\frac{\text{area}(\triangle ABC)}{\text{area}(\triangle DEF)} = \left(\frac{AB}{DE}\right)^2 = \frac{9}{64}$$

$$\left(\frac{AB}{DE}\right)^2 = \frac{9}{64}$$

$$\Rightarrow \frac{AB}{DE} = \frac{3}{8}$$

$$\frac{AB}{5.1} = \frac{3}{8}$$

$$\Rightarrow AB = \frac{3 \times 5.1}{8}$$

$$\boxed{AB = 1.9125\ cm}$$

iii) $$\frac{area\,(\Delta ABC)}{area\,(\Delta DEF)} = \left(\frac{AC}{DF}\right)^2 = \left(\frac{19}{8}\right)^2$$

$$\Rightarrow \boxed{\frac{area(\Delta ABC)}{area\,(\Delta DEF)} = \frac{361}{64}}$$

The ratio of areas of two triangles is 361:64.

iv)

$$\frac{area\,(\Delta ABC)}{area\,(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2 = \frac{36}{64}$$

$$\Rightarrow \left(\frac{AB}{DE}\right)^2 = \left(\frac{6}{8}\right)^2$$

$$\Rightarrow \frac{AB}{DE} = \frac{6}{8}$$

$$\Rightarrow \boxed{AB = 4.65\ cm}$$

v) $$\frac{Area\,(\Delta ABC)}{Area\,(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{1.2}{1.4}\right)^2$$

$$\frac{area\,(\Delta ABC)}{area\,(\Delta DEF)} = \frac{36}{49}$$

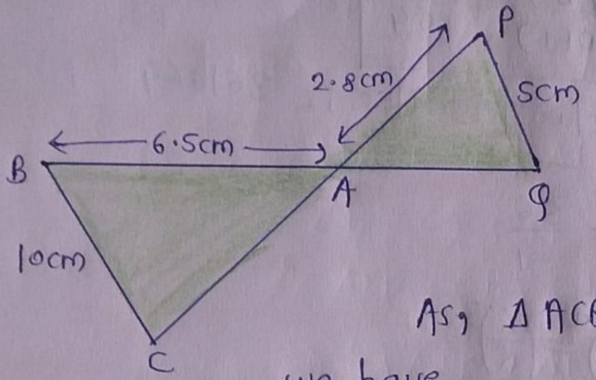
Thus, the ratio of the areas of the two triangles are 36:49.

Q.2) In the fig. 4.178, $\Delta ACB \sim \Delta APQ$, If $BC = 10cm$, $PQ = 5cm$, $BA = 6.5cm$, $AP = 2.8cm$, find CA and AQ. Also find the area (ΔACB) : area (ΔAPQ).

Soln:— Here, given that $\Delta ACB \sim \Delta APQ$,

$BC = 10cm$, $PQ = 5cm$, $BA = 6.5cm$, $AP = 2.8cm$

To find CA, AQ and $\dfrac{\text{area}(\Delta ACB)}{\text{area}(\Delta APQ)}$:



As, $\Delta ACB \sim \Delta APQ$,

we have,

According to corresponding parts of similar triangles are equal, we can write

$$\frac{AB}{AQ} = \frac{BC}{PQ} = \frac{AC}{AP}$$

$$\frac{AB}{AQ} = \frac{BC}{PQ}$$

$$\frac{6.5}{AQ} = \frac{10}{5}$$

$$\Rightarrow \boxed{AQ = 3.25cm}$$

In similar way, $\dfrac{BC}{PQ} = \dfrac{CA}{AP}$

$$\frac{CA}{2.8} = \frac{10}{5}$$

$$\Rightarrow \boxed{CA = 5.6cm}$$

As, the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides,

we can write,

$$\frac{\text{area}(\Delta ACQ)}{\text{area}(\Delta APQ)} = \left(\frac{BC}{PQ}\right)^2$$

$$= \left(\frac{10}{5}\right)^2$$

$$= \left(\frac{2}{1}\right)^2$$

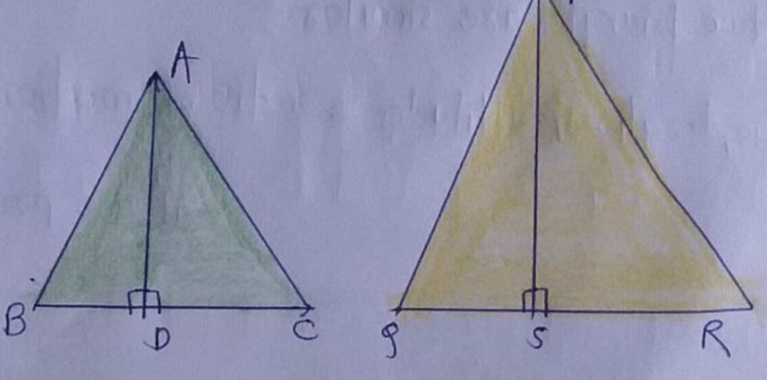
$$\boxed{\frac{\text{area}(\Delta ACQ)}{\text{area}(APQ)} = \frac{4}{1}}$$

Thus, the ratio of areas of two triangles is 4:1.

Q.3. The areas of two similar triangles are $81\,cm^2$ and $49\,cm^2$ respectively. Find the ratio of their corresponding heights. What is the ratio of their corresponding medians?

Soln:- Here, given that the areas of two similar triangles is $81\,cm^2$ and $49\,cm^2$.

To find the ratio of their corresponding heights and the ratio of their corresponding medians:



Now, we consider two similar triangles namely ABC & PQR respectively as shown in fig.

In ΔABC and ΔPQR, AD and PS are the altitudes of ΔABC and ΔPQR respectively.

Thus, According to similar triangle theorem,

we can write,

$$\frac{area(\Delta ABC)}{area(\Delta PQR)} = \frac{(AB)^2}{(PQ)^2}$$

$$\Rightarrow \frac{81}{49} = \frac{AB^2}{PQ^2}$$

$$\Rightarrow \boxed{\frac{AB}{PQ} = \frac{9}{7}}$$

Now, in ΔABD and ΔPQS,

$\angle B = \angle Q$ $\quad \because \Delta ABC \sim \Delta PQR$

$\angle ABD = \angle PQ = 90°$

$\Rightarrow \boxed{\Delta ABD \sim \Delta PQS}$ $\quad \because$ by AA

Since, if the two triangles are similar then their corresponding parts are proportional.

we can write,

$$\frac{AB}{PQ} = \frac{AD}{PS}$$

But, $\boxed{\frac{AD}{PS} = \frac{9}{7}}$ $\longrightarrow$, is the ratio of heights

So, the ratio of altitudes is equal to ratio of medians if two triangles are similar.

Thus, ratio of altitudes = ratio of medians = 9/7

Hence proved.

Q. 4) The areas of two similar triangles are 169 cm² and 121 cm² respectively. If the longest side of the larger triangle is 26 cm, find the longest side of the smaller triangle.

Solⁿ:- Here, given that

The two similar triangles are having areas 169 cm² and 121 cm².

The larger triangle is having largest side whose length is 26 cm.

To find the length of longest side of smaller triangle:

Let us consider the length of longer side of smaller triangle is x.

But, we already know that, the ratio of areas of two similar triangles is equal to ratio of their squares of corresponding sides.

$$\Rightarrow \frac{\text{area (larger triangle)}}{\text{area (smaller triangle)}} = \frac{(\text{side of larger triangle})^2}{(\text{side of smaller triangle})^2}$$

$$= \frac{169}{121}$$

- on taking square roots $\Rightarrow$ $\frac{13}{11} = \frac{\text{side of larger } \Delta}{\text{side of smaller } \Delta}$

But, the sides of similar triangle are proportional.

$$\Rightarrow \frac{13}{11} = \frac{\text{longer side of larger } \Delta}{\text{longer side of smaller } \Delta}$$

$$\frac{13}{11} = \frac{26}{x}$$

$$13x = 26 \times 11$$

$$\boxed{x = 22 \text{ cm}}$$

Thus, the longest side of the smaller triangle is 22 cm.

Q.5.) The area of two similar triangles are $25 cm^2$ and $36 cm^2$ respectively. If the altitudes of the first triangle is 2.4 cm, find the corresponding altitude of the other.

Soln:- Here, given that

The area of two similar triangle is $25 cm^2$ and $36 cm^2$ respectively.

And the altitude of first triangle is 2.4 cm.

To find the altitude of second triangle:

But, we already know that,

The ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding altitudes.

Thus,

$$\frac{\text{area (triangle 1)}}{\text{area (triangle 2)}} = \frac{(\text{altitude 1})^2}{(\text{altitude 2})^2}$$

$$\frac{25}{36} = \frac{(2.4)^2}{(\text{altitude})^2}$$

on taking square root on both sides,

$$\frac{5}{6} = \frac{2.4}{\text{altitude 2}}$$

$$\Rightarrow (\text{altitude 2}) = (2.4 \times 6)/5 = 2.88 cm$$

Thus, the altitude of the second triangle is found to be 2.88 cm.

Q.6.) The corresponding altitudes of two similar triangles are 6cm and 9cm respectively. Find the ratio of their areas.

Soln:- Here, given that

The two triangles are similar which are having corresponding altitudes as 6cm and 9cm respectively.

To find the ratio of areas of two similar triangles:

But, we already know that,
the ratio of areas of two similar triangles is equal to ratio of squares of their corresponding sides.

$$\Rightarrow \frac{\text{area (triangle 1)}}{\text{area (triangle 2)}} = \frac{(\text{altitude 1})^2}{(\text{altitude 2})^2}$$

$$= \frac{36}{81}$$

Taking square root on both sides,

$$\frac{\text{altitude 1}}{\text{altitude 2}} = \frac{6}{9}$$

Thus, the ratio of the areas of two triangles $= 36:81$

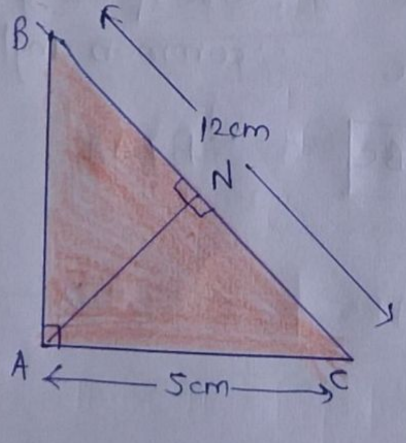
Q.7.) ABC is a triangle in which $\angle A = 90°$, AN $\perp^{er}$ BC, BC = 12 cm and AC = 5cm. Find the ratio of the areas of ΔANC and ΔABC.

Sol^n:– Here, given that

$\angle A = 90°$, AN $\perp^{er}$ BC in ΔABC.

BC = 12 cm, AC = 5 cm

To find $\frac{\text{area}(\Delta ANC)}{\text{area}(\Delta ABC)}$:



Here, we have
In ΔANC & ΔABC,

$\angle ACN = \angle ACB$ $\because$ common angle
$\angle A = \angle ANC = 90°$

$\Rightarrow \boxed{\Delta ANC \sim \Delta ABC}$ $\because$ By AA

But, the ratio of areas of two similar triangle is equal to the ratio of squares of their corresponding sides.

So, we can write,

$$\frac{\text{area}(\Delta ANC)}{\text{area}(\Delta ABC)} = \frac{(AC)^2}{(BC)^2}$$

$$= \frac{5^2}{12^2}$$

$$= \frac{25}{144}$$

Thus, the ratio of areas of two triangle = 25:144.

Q.8.) In fig. 4.179, DE ∥ BC

i) If DE = 4m, BC = 6cm and area (ΔADE) = 16cm^2, find the area of ΔABC.

ii) If DE = 4cm, BC = 8cm and area (ΔADE) = 25cm^2, find the area of ΔABC.

iii) If DE : BC = 3:5, Calculate the ratio of the areas of ΔADE and the trapezium BCED.

Soln:- Here, given that DE ∥ BC.

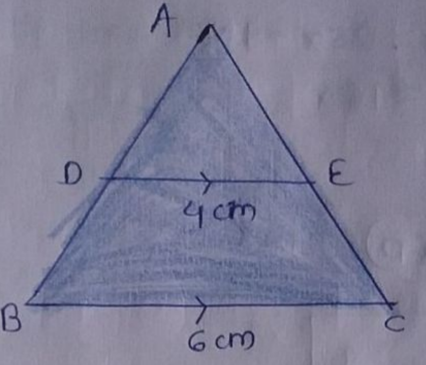
In ΔADE and ΔABC,

we have

$\angle ADE = \angle B$ ∵ corresponding angle

$\angle DAE = \angle BAC$ ∵ common angle

Thus, $\boxed{\Delta ADE \sim \Delta ABC}$ ∵ BY AA



i) we know that,
the ratios of areas of two similar triangle is equal to the ratio of squares of their corresponding sides.

$$\frac{\text{Area}(\Delta ADE)}{\text{Area}(\Delta ABC)} = \frac{DE^2}{BC^2}$$

$$\frac{16}{\text{Area}(\Delta ABC)} = \frac{4^2}{6^2}$$

$$\Rightarrow \text{Area}(\Delta ABC) = (8^2 \times 25)/4^2$$

$$\Rightarrow \boxed{\text{Area}(\Delta ABC) = 100\ cm^2}$$

ii) According to the given condition,

$$\frac{\text{Area}(\Delta ADE)}{\text{Area}(\Delta ABC)} = \frac{DE^2}{BC^2}$$

$$= \frac{3^2}{5^2}$$

$$\frac{\text{Area}(\Delta ADE)}{\text{Area}(\Delta ABC)} = \frac{9}{25}$$

Now, we consider $A(\Delta ABC) = 25x$ sq. units
and $A(\Delta ADE) = 9x$ sq. units

Then
Area of trapezium BCED = Area of ΔABC − Area of ΔADE

Area of trapezium BCED $= 25x - 9x$
$= 16x$

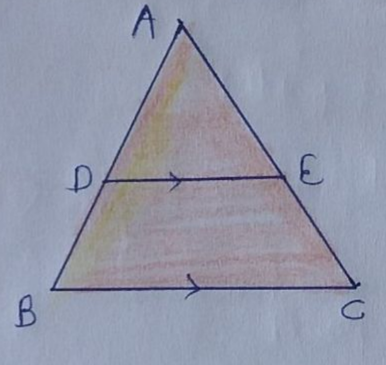
Thus, $\frac{\text{Area}(\Delta ADE)}{\text{Area (trap. BCED)}} = \frac{9x}{16x}$

$$\boxed{\frac{\text{Area}(\Delta ADE)}{\text{Are (trap. BCED)}} = \frac{9}{16}}$$

Q.9.) In ΔABC, D and E are the mid-points of AB and AC respectively. Find the ratio of the areas of ΔADE and ΔABC.

Soln:- Here, given that
In ΔABC, D and E are the midpoints of side AB and side AC respectively.

To find the ratios of areas of ΔADE & ΔABC:



we have,
D & E are midpoints of AB & AC respe.
$\Rightarrow DE \parallel BC$
(since according to converse of mid-point theorem)

Again, $DE = \frac{1}{2}(BC)$

In ΔADE & ΔABC,
$\angle ADE = \angle B$ ∵ corresponding angles
$\angle DAE = \angle BAC$ ∵ common angle

Thus, $\boxed{\Delta ADE \sim \Delta ABC}$ ∵ by AA

But, we already know that

The ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides,

$$\frac{\text{Area}(\Delta ADE)}{\text{Area}(\Delta ABC)} = \frac{(AD)^2}{(AB)^2} = \frac{1^2}{2^2} = \frac{1}{4}$$

$$\text{Area}(\Delta ADE) : \text{Area}(\Delta ABC) = 1:4$$

Q.10.) The areas of two similar triangles are $100cm^2$ and $49cm^2$ respectively. If the altitude of the bigger triangles is 5 cm, find the corresponding altitude of the other.

Soln:- Here, given that

The areas of two similar triangles are $100cm^2$ and $49\ cm^2$ respectively.

The altitude of bigger triangle is 5cm.

To find the corresponding altitude of the other triangle:

we have,

The ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

$$\Rightarrow \frac{\text{Area (Bigger triangle)}}{\text{Area (smaller triangle)}} = \frac{(\text{altitude of bigger triangle})}{(\text{altitude of smaller triangle})}$$

$$\frac{100}{49} = \frac{5^2}{(\text{altitude of smaller triangle})^2}$$

Taking square root on both sides,

$$\frac{10}{7} = \frac{5}{\text{altitude of smaller triangle}}$$

$$\Rightarrow \text{altitude of smaller triangle} = \frac{7 \times 5}{10}$$

Therefore, the altitude of smaller triangle 3.5 cm.

Q.11.) The areas of two similar triangles are $121 cm^2$ and $64 cm^2$ respectively. If the median of the first tringle is 12.1 cm, find the corresponding median of the other.

Soln:-

Here, given that

The area of two similar triangles are $121 cm^2$ and $64 cm^2$ respectively.

The median of the first triangle is 12.1 cm.

To find the corresponding median of the other triangle:

We already know that,

The ratio of areas of two similar triangles are equal to the ratio of the squares of their medians.

$$\Rightarrow \frac{\text{Area (triangle 1)}}{\text{Area (triangle 2)}} = \frac{(\text{median of triangle 1})^2}{(\text{median of triangle 2})^2}$$

$$\frac{121}{64} = \left(\frac{12.1}{\text{median of } \Delta 2}\right)^2$$

Taking square root on both sides,

$$\frac{11}{8} = \left(\frac{12.1}{\text{median of } \Delta 2}\right)$$

Thus, median of triangle 2 = $8 \times 12.1/11$

Median of triangle 2 = 8.8 cm.

Exercise 47

Q.1) If the sides of a triangle are 3cm, 4cm and 6cm long, determine whether the triangle is a right-angled triangle.

Soln:- Here, given that

In ΔABC, $AB = 3cm$, $BC = 4cm$, $AC = 6cm$

Squares of sides $\Rightarrow$

$(AB)^2 = 9$, $(BC)^2 = 16$, $(AC)^2 = 36$

Since, here $AB^2 + BC^2 \neq AC^2$

$9 + 16 \neq 36$

Thus, by the converse of Paythagoras theorem the given sides cannot form the sides of right-angled triangle.

Q.2) The sides of certain triangles are given below. Determine which of them are right triangles.

i) $a = 7cm$, $b = 24cm$ and $c = 25cm$

ii) $a = 9cm$, $b = 16cm$ and $c = 18cm$

iii) $a = 1.6cm$, $b = 3.8cm$, $c = 4cm$

iv) $a = 8cm$, $b = 10cm$, $c = 6cm$

Soln:- i) Here, given that

$a = 7cm$, $b = 24cm$, $c = 25cm$

$\therefore a^2 = 49$, $b^2 = 576$, $c^2 = 625$

Now, $a^2 + b^2 = 49 + 576 = 625 = c^2$

Thus, $\underline{a^2 + b^2 = c^2}$

Thus, by converse of Paythagoras theorem,

We can say that,

The given sides forms the right-angled triangle.

ii) Here, given that

$a = 9cm$, $b = 16cm$ and $c = 18cm$

$\Rightarrow a^2 = 81$, $b^2 = 256$, $c^2 = 6324$

Thus, $a^2 + b^2 = 81 + 256 = 337 \neq c^2$

Thus, $\underline{a^2 + b^2 \neq c^2}$

Then, according to converse of Paythagoras theorem, the given sides cannot forms the right-angled triangle here.

iii) Here, given that

~~a = 9 cm~~ $a = 1.6cm$, $b = 3.8cm$, $c = 4cm$

$\Rightarrow a^2 = 2.56$, $b^2 = 14.44$, $c^2 = 16$

Since, here $a^2 + b^2 = 2.56 + 14.44 = 17 \neq c^2$

Thus, $\underline{a^2 + b^2 \neq c^2}$

According to converse of paythagoras theorem, the given sides cannot form the right-angled triangle.

iv) Here, given that

$a = 8cm$, $b = 10cm$, $c = 6cm$

$\Rightarrow a^2 = 64$, $b^2 = 100$, $c^2 = 36$

Thus, $a^2 + b^2 = 64 + \cancel{100}^{36} = \cancel{100 = b^2}$

~~Hence, $a^2 + b^2 =$~~

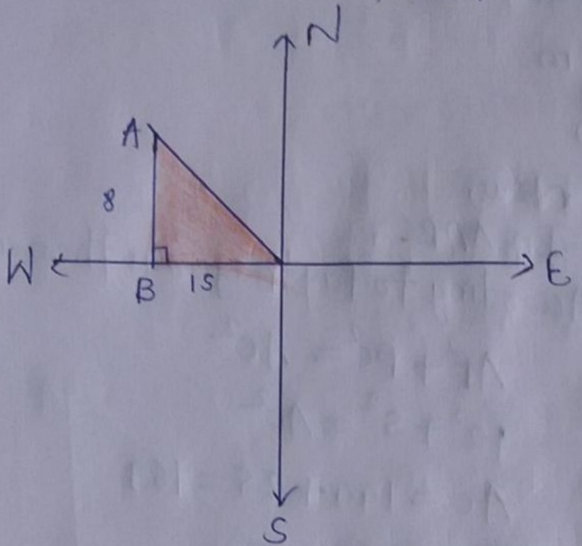
Thus, $a^2 + c^2 = 64 + 36 = 100 = b^2$

$\Rightarrow \underline{a^2 + c^2 = b^2}$

Then, according to converse of Paythagoras theorem, the given sides forms the right-angled triangle.

Q.3) A man goes 15 meters due west and then 8 meters due north. How far is from the starting point?

Soln:- Let us consider, the starting point of man is 'O' and it's final point is A.



Now, In ΔABO,

According to Paythagoras theorem

Since $\angle ABO = 90^\circ$

$AO^2 = AB^2 + BO^2$

$AO^2 = 8^2 + 15^2$

$AO^2 = 64 + 225 = 289$

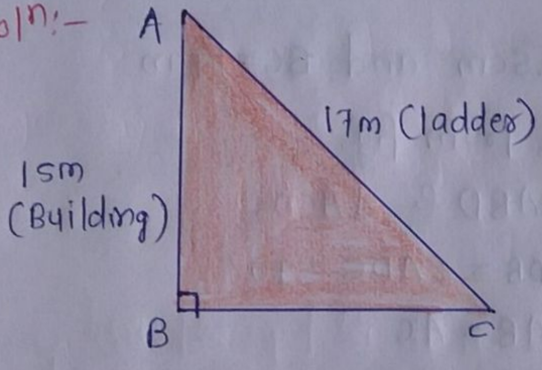
$\Rightarrow AO = \sqrt{289}$

$\boxed{AO = 17m}$

Thus, the man is 17m far from the starting point.

Q.4) A ladder 17m long reaches a window of a building 15m above the ground. Find the distance of the foot of the ladder from the building.

Soln:-



In ΔABC,

$\angle ABC = 90^\circ$

Then, according to Paythagoras theorem,

$AB^2 + BC^2 = AC^2$

$15^2 + BC^2 = 17^2$

$225 + BC^2 = 17^2$

$BC^2 = 289 - 225$

$BC^2 = 64 \Rightarrow \boxed{BC = 8m}$

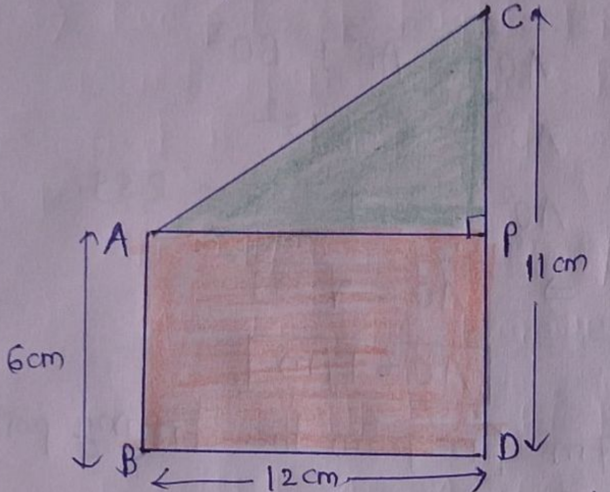
Thus, the distance of the foot of the ladder from building = 8m.

Q.5) Two poles of heights 6m and 11m stand on a plane ground. If the distance between their feet is 12m, find the distance between their tops.

Soln:- From fig. consider that CD and AB be the poles of height 11m and 6m.

Then, $CP = 11 - 6 = 5m$.

But, from fig. $AP = 12m$



Now,

In ΔAPC,

According to Paythagoras theorem,

$AP^2 + PC^2 = AC^2$

$12^2 + 5^2 = AC^2$

$AC^2 = 144 + 25 = 169$

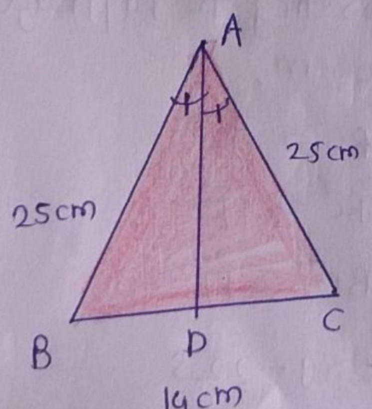
$\boxed{AC = 13}$

Thus, the distance between the tops of poles is found to be 13m.

Q.6) In an isosceles triangle ABC, $AB = AC = 25cm$, $BC = 14cm$. Calculate the altitude from A on BC.

Soln:- Here, given that

In ΔABC, $AB = AC = 25cm$ and $BC = 14cm$



Now, from fig.

In ΔABD & ΔACD,

$\angle ADB = \angle ADC = 90°$

$AB = AC$

$AD = AD$ ∵ common side

$\Rightarrow \boxed{\Delta ABD \cong \Delta ACD}$

Then, BD = CD = 7cm

Since, corresponding parts of congruent triangles are equal.

Now, In ΔADB,

According to Paythagoras theorem,

$$AD^2 + BD^2 = AB^2$$

$$AD^2 + 7^2 = 25^2$$

$$\therefore AD^2 = 625 - 49 = 576$$

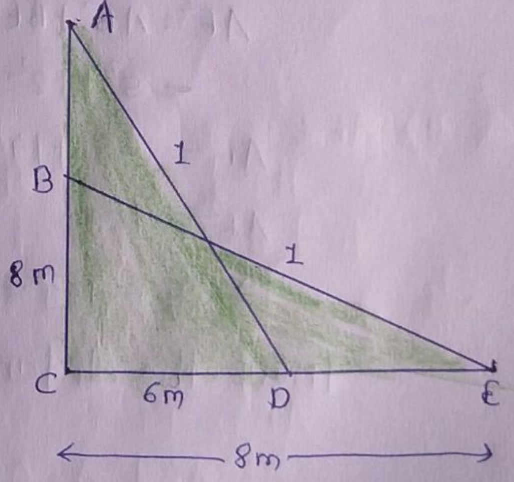
$$AD = \sqrt{576} = 24 \text{ cm}$$

$$\boxed{AD = 24cm}$$

Q.7) The foot of a ladder is 6m away from a wall and its top reaches a window 8m above the ground. If the ladder is shifted in such a way that its foot is 8m away from the wall, to what height does its tip reach?

Soln:- let us consider the length of ladder,

$$AD = BE = x \text{ m}$$



In ΔACD,

By Paythagoras theorem,

$$AD^2 = AC^2 + CD^2$$

$$\Rightarrow x^2 = 8^2 + 6^2 \text{ --- (1)}$$

Now, In ΔBCE,

By Paythagoras theorem,

$$BE^2 = BC^2 + CE^2$$

$$x^2 = BC^2 + 8^2 \text{ --- (2)}$$

comparing (1) and (2),

$$BC^2 + 8^2 = 8^2 + 6^2$$

$BC^2 = 6^2$

$\boxed{BC = 6m}$

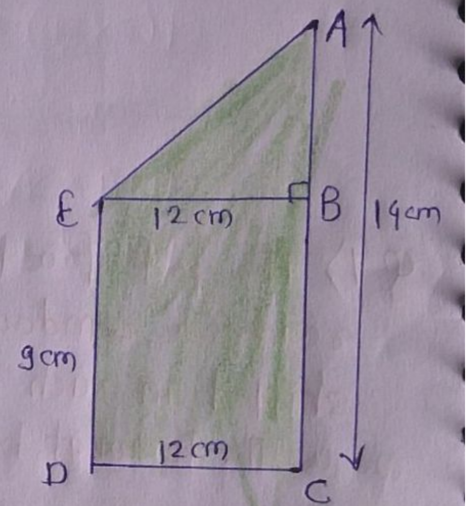
Thus, the tip of the ladder reaches to a height of 6m.

Q.8) Two poles of height 9 m and 14m stand on a plane ground. If the distance between their feet is 12m, find the distance between their ~~go~~ tops.

Soln:-

From figure, we can say that

$AC = 14m$, $DC = 12m$ and $ED = BC = 9m$



Now, we will draw $EB \perp^{er} AC$.

From that, $AB = AC - BC$

$= (14 - 9)$

$= 5m$

And also, $EB = DC = 12m$

Now, In ΔABE, by Paythagoras theorem,

$AE^2 = AB^2 + BE^2$

$= 5^2 + 12^2$

$AE^2 = 25 + 144$

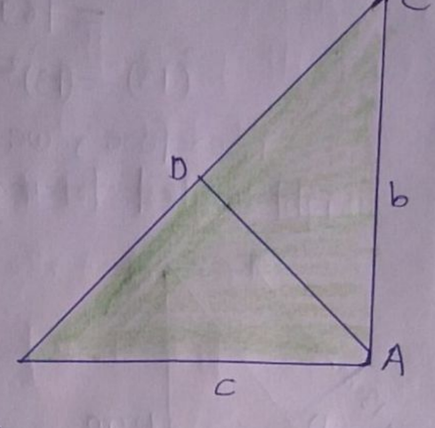
$AE^2 = 169$

$\Rightarrow AE = \sqrt{169} = 13m$

Thus, the distance between their tops $= 13m$.

Q.9.) Using Paythagoras theorem determine the length of AD in terms of b and c shown in fig. 4.219.

Soln:- Here, In ΔABC,



According to Paythagoras theorem,

$$BC^2 = AB^2 + AC^2$$

$$BC^2 = c^2 + b^2$$

$$BC = \sqrt{c^2 + b^2}$$

Now, In ΔABD & ΔCBA,

$\angle B = \angle B$ $\because$ common angle

$\angle ADB = \angle BAC = 90^\circ$

$\Rightarrow$ $\boxed{\Delta ABD \sim \Delta CBA}$ $\because$ By AA

Thus, we can write according to corresponding parts of similar triangles are proportional.

$$\frac{AB}{CB} = \frac{AD}{CA}$$

$$\frac{c}{\sqrt{(c^2+b^2)}} = \frac{AD}{b}$$

$$\boxed{\therefore AD = bc/\sqrt{b^2+c^2}}$$

Q.10.) A triangle has sides 5cm, 12cm and 13cm. Find the length to one decimal place, of the perpendicular from the opposite vertex to the side whose length is 13cm.

Soln:- From given, we can write

$AB = 5\text{ cm}$, $BC = 12\text{ cm}$, $AC = 13\text{ cm}$

Then by Paythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

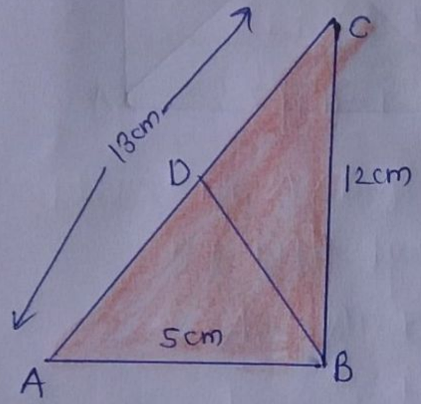
$\Rightarrow (13)^2 = (5)^2 + (12)^2$

$= 25 + 144$

$= 169$

$(13)^2 = (13)^2$

Therefore, we can say that the triangle ABC is a right angled triangle at point B.



Let us consider, BD is the ⊥er drawn along side AC as shown in fig.

Now, Area $(\Delta ABC) = (BC \times BA)/2$

$= (12 \times 5)/2$

$= 30\ cm^2$

Again area $(\Delta ABC) = (AC \times BD)/2$

$= (13 \times BD)/2$

Now, we will write,

$$\frac{(13 \times BD)}{2} = 30$$

$13 \times BD = 60$

$$\boxed{BD = 60/13 = 4.6}$$

Q.11.) ABCD is a square. F is the midpoint of AB. BE is one third of BC. If the area of $\Delta FBE = 108\ cm^2$, find the length of AC.

Soln:- Here, given that

ABCD is a square, where F is the midpoint of AB.

Also, BE is one third of BC here.

And $A(\Delta FBE) = 108\ cm^2$

To find length of AC:

Let us consider the side of square is 'x'.

$\Rightarrow AB = BC = CD = DA = x$ cm

also, $AF = FB = x/2$ cm

Thus, $BE = x/3$ cm

Now, Area of $\Delta FBE = \frac{1}{2} \times BE \times FB$

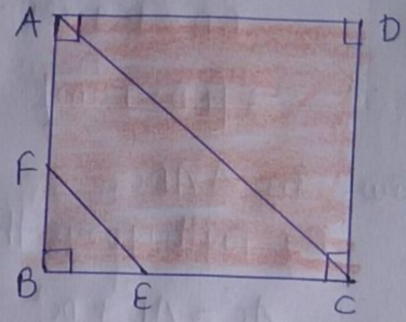
$$108 = (1/2) \times (x/3)(x/2)$$

$$x^2 = 108 \times 2 \times 3 \times 2$$

$$x^2 = 1296$$

$$x = \sqrt{1296}$$

$$\therefore \boxed{x = 36 \text{ cm}}$$



Now, by Paythagoras theorem,

In ΔABC, $AC^2 = AB^2 + BC^2$

$$AC^2 = x^2 + x^2$$

$$AC^2 = 2x^2$$

$$AC^2 = 2(36)^2$$

$$AC = 36\sqrt{2} = 36 \times 1.414$$

$$\boxed{AC = 50.904 \text{ cm}}$$

Thus, here the length of AC is found to be 50.904 cm.

Q.12.) In an isosceles triangle ABC, if $AB = AC = 13$cm and the altitude from A on BC is 5cm, find BC.

Soln:- Here given that,

In an isosceles triangle ABC,

$$AB = AC = 13 \text{ cm}$$

And altitude from A on BC is $AD = 5$ cm.

To find BC:

From fig,

In ΔADB,

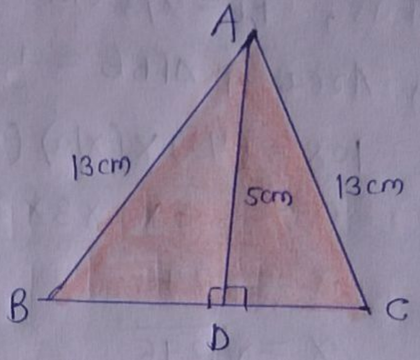
By Paythagoras theorem,

$AD^2 + BD^2 = 13^2$

$5^2 + BD^2 = 169$

$BD^2 = 169 - 25 = 144$

$\Rightarrow \boxed{BD = 12cm}$



Now, In ΔADC,

By Paythagoras theorem,

$AC^2 = AD^2 + DC^2$

$13^2 = 5^2 + DC^2$

$DC^2 = 169 - 25$

$DC^2 = 144$

$\boxed{DC = 12cm}$

Thus, here $BC = BD + DC = 12 + 12 = 24cm$

Q.13.) In a ΔABC, $AB = BC = CA = 2a$ and $AD \perp^{er} BC$, Prove that

i) $AD = a\sqrt{3}$

ii) Area $(\Delta ABC) = \sqrt{3}a^2$

Sol^n:- i) Here, given that

In ΔABC, $AB = BC = CA = 2a$

also, $AD \perp^{er} BC$ and $AD = a\sqrt{3}$

Now, from fig,

In ΔABD & ΔACD,

$\angle ADB = \angle ADC = 90°$

$AB = AC$

$AD = AD$ $\quad \because$ common side

$\Rightarrow \boxed{\Delta ABD \cong \Delta ACD}$

Hence, we can write, $BD = CD = a$

Now, in ΔABD,

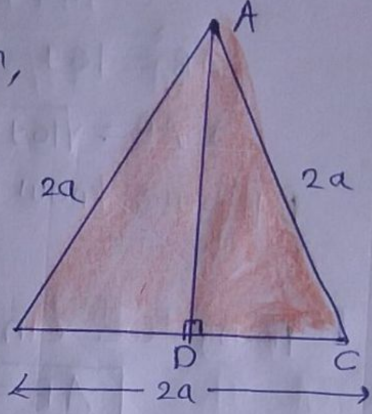
According to Paythagoras theorem,

$AD^2 + BD^2 = AB^2$

$AD^2 + a^2 = (2a)^2$

$AD^2 = 4a^2 - a^2 = 3a^2$

$\boxed{AD = \sqrt{3}a}$



ii) Here, given that,

In ΔABC, $AB = BC = CA = 2a$ and $AD \perp^{er} BC$.

Also, $A(\Delta ABC) = \frac{1}{2} \times BC \times AD$

$= 1/2 \times (2a) \times (a\sqrt{3})$

$\boxed{A(\Delta ABC) = \sqrt{3}a^2}$

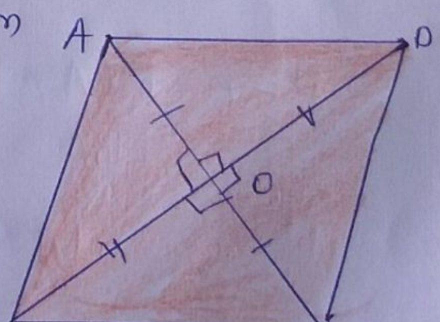
Q.14.) The length of the diagonals of a rhombus is 24cm and 10cm. Find each side of the rhombus.

Soln:— Let us consider ABCD is a rhombus.

And ~~sir~~ AC, BD are the diagonals of ABCD.

So, from given condition we can write,

$AC = 24\text{cm}$, $BD = 10\text{cm}$



we already know that,

the diagonals of a rhombus bisects each other at right angle only.

Thus, $AO = OC = 12\text{cm}$

$BO = OD = 3\text{cm}$

In ΔAOB,

According to Paythagoras theorem,

$$AB^2 = AO^2 + BO^2$$
$$= 12^2 + 5^2$$
$$= 144 + 25$$
$$AB^2 = 169$$
$$\Rightarrow AB = \sqrt{169} = 13cm$$

But, we know that, all the sides of a rhombus are equal already.

Thus, we can write here

$$\boxed{AB = BC = CD = AD = 13cm}$$