* Class 10^{th} – Math Solutions *

Chapter 1: Real Numbers

Exercise 1.1

Q.1.) If a and b are two odd positive integers such that $a>b$, then prove that one of the two numbers $(a+b)/2$ & $(a-b)/2$ is odd and the other is even.

Solⁿ:- • We know that, odd number is that number which is not divisible by 2.

• Hence, odd number can be written as $(4q+1)$ or $(4q+3)$, where q is a some whole number.

• Here, given that $a>b$, Hence we can write these odd numbers as $a=4q+3$ and $b=4q+1$, since $a>b$

• Now, $\frac{(a+b)}{2} = \frac{[(4q+3)+(4q+1)]}{2} = \frac{(8q+4)}{2} = 4q+2$

$(a+b)/2 = 4q+2 = 2(2q+1)$ is the even number only.

Since any number multiplied by 2 is always an even number.

• Now, $\frac{(a-b)}{2} = \frac{[(4q+3)-(4q+1)]}{2} = \frac{2}{2} = 1$ is an odd number.

Thus, $\frac{(a+b)}{2}$ is an even number & $\frac{(a-b)}{2}$ is an odd number.

Q.2) Prove that, the product of consecutive positive integers is divisible by 2.

Soln:- • To prove, product of two consecutive positive integers is divisible by 2.

• Consider two consecutive positive integers as $(n-1)$ & n.

• Now, the product of two consecutive positive integers.

Here $\boxed{(n-1)n = n^2 - n}$

• From, Euclid's division lemma for $b=2$

we have, any positive integer is of the form $2q$ or $(2q+1)$.

• Thus, when $n = 2q$

we have, $n^2 - n = (2q)^2 - (2q)$

$= 4q^2 - 2q$

$\boxed{n^2 - n = 2(2q^2 - q)}$

Here, (n^2-n) is divisble by 2 when $n = 2q$.

Now, when $n = 2q+1$

we have, $n^2 - n = (2q+1)^2 - (2q+1)$

$= (4q^2 + 1 + 4q) - 2q - 1$

$= 4q^2 + 2q$

$\boxed{n^2 - n = 2(2q^2 + q)}$

Here, $(n^2 - n)$ is also divisible by 2 when $n = 2q+1$.

Thus, it is proved that, the product of two consecutive positive integers is divisible by 2 always.

Q.3) Prove that, the product of three consecutive positive integers is divisible by 6.

Soln:- • To prove, the product of three consecutive positive integers is divisible by 6.

• Consider any positive integer suppose, n.

• Then, the three consecutive integers will be: $n, n+1, n+2$

• But, we allready know that, any positive integer can be written in the form $6q$ or $(6q+1)$ or $(6q+2)$ or $(6q+3)$ or $(6q+4)$ or $(6q+5)$

According to Euclid's division lemma for $b=6$.

• Now,

when $n = 6q$

Product of 3 consecutive +ve integers $= n(n+1)(n+2)$

$\Rightarrow 6q(6q+1)(6q+2)$

$\Rightarrow 6[q(6q+1)(6q+2)]$

$\Rightarrow \boxed{n(n+1)(n+2) = 6m}$ $\because m = q(6q+1)(6q+2)$

is divisible by 6.

• when $n = 6q+1$

$\Rightarrow (n)(n+1)(n+2) = (6q+1)(6q+2)(6q+3)$

$= 6[(6q+1)(3q+1)(2q+1)]$

$\boxed{n(n+1)(n+2) = 6m}$ $\because m = (6q+1)(3q+1)(2q+1)$

is divisible by 6

• when $n = 6q+2$

$n(n+1)(n+2) = (6q+2)(6q+3)(6q+4)$

$= 6[(3q+1)(2q+1)(6q+4)]$

$\boxed{n(n+1)(n+2) = 6m}$ $\because m = (3q+1)(2q+1)(6q+4)$

is divisible by 6.

• when $n = 6q+3$

$\Rightarrow n(n+1)(n+2) = (6q+3)(6q+4)(6q+5)$

$= 6[(3q+1)(3q+2)(6q+5)]$

$\boxed{n(n+1)(n+2) = 6m}$ $\because m = (3q+1)(3q+2)(6q+5)$

↳ is divisible by 6.

• when $n = 6q+4$

$\Rightarrow n(n+1)(n+2) = (6q+4)(6q+5)(6q+6)$

$= 6[(3q+2)(3q+1)(2q+2)]$

$\boxed{n(n+1)(n+2) = 6m}$ $\because m = (3q+2)(3q+1)(2q+2)$

↳ is divisible by 6.

• when $n = 6q+5$

$\Rightarrow n(n+1)(n+2) = (6q+5)(6q+6)(6q+7)$

$= 6[(6q+5)(q+1)(6q+7)]$

$\boxed{n(n+1)(n+2) = 6m}$ $\because m = (6q+5)(q+1)(6q+7)$

↳ is divisible by 6.

In this way, the product of three consecutive positive integers is divisible by 6.

Q.4) For any positive integer n, prove that (n^3-n) is divisible by 6.

$\rightarrow$ To prove any positive integer n, (n^3-n) is divisible by 6.
But, we know that, any positive integer in the form $(6q+2)$, or $(6q+3)$ or $(6q+4)$ or $(6q+5)$
According to Euclid's division lemma for $b=6$.

we have, $n^3-n = n(n^2-1) = n(n-1)(n+1) = (n-1)n(n+1)$

$$\boxed{n^3-n = (n-1)n(n+1)}$$

- when $n = 6q$,

$(n-1)n(n+1) = (6q-1)6q(6q+1)$

$= 6[(6q-1)q(6q+1)]$

$\boxed{(n-1)n(n+1) = 6m}$ is divisible by 6. $\because m = (6q-1)q(6q+1)$

- when $n = 6q+1$,

$\Rightarrow (n-1)n(n+1) = (6q)(6q+1)(6q+2)$

$= 6[q(6q+1)(6q+2)]$ $\because m = q(6q+1)(6q+2)$

$\boxed{(n-1)n(n+1) = 6m}$ is divisible by 6.

- when $n = 6q+2$,

$\Rightarrow (n-1)n(n+1) = (6q+1)(6q+2)(6q+3)$

$= 6[(6q+1)(3q+1)(2q+1)]$

$\boxed{(n-1)n(n+1) = 6m}$ is divisible by 6. $\because m = (6q+1)(3q+1)(2q+1)$

$(6q+7)$

• when $\underline{n = 6q+3}$,

$$\Rightarrow (n-1)\,n(n+1) = (6q+3)(6q+4)(6q+4)$$
$$= 6[(2q+1)(3q+1)(6q+4)]$$

$\boxed{(n-1)n(n+1) = 6m}$ is divisible by 6 $\because m = (2q+1)(3q+1)(6q+4)$

• when $n = 6q+4$,

$$\Rightarrow (n-1)\,n(n+1) = (6q+3)(6q+4)(6q+5)$$
$$= 6[(2q+1)(3q+2)(6q+5)]$$

$(n-1)n(n+1) = 6m$ which is divisible by 6

$\because m = (2q+1)(3q+2)(6q+5)$

• when $n = 6q+5$,

$$\Rightarrow (n-1)(n)(n+1) = (6q+4)(6q+5)(6q+6)$$
$$= 6[(6q+4)(6q+5)(q+1)]$$

$\boxed{(n-1)\,n(n+1) = 6m}$ is divisible by 6. $\because m = (6q+4)(6q+5)(q+1)$

Thus, the product of three consecutive positive integers is divisible by 6.

Q.4) For any positive integer n, prove that (n^3-n) divisible by 6.

→ • Let us consider 'n' be any positive integer.
According to Euclid's division formula/lemma for $b=6$,
• Any positive integer is of the form $6q$ or $6q+1$ or $6q+2$ or $6q+3$ or $6q+4$ or $6q+5$.

• To prove (n^3-n) is divisible by 6.

$$(n^3-n) = n(n^2-1) = (n-1)\,n\,(n+1)$$

• When $n=6q$

$$(n-1)\,n\,(n+1) = (6q-1)(6q)(6q+1)$$
$$= 6[(6q-1)q(6q+1)]$$

$\boxed{(n-1)\,n\,(n+1) = 6m}$ is divisible by 6. $\because m = (6q-1)q(6q+1)$

• when $n = 6q+1$,

$$(n-1)\,n\,(n+1) = (6q)(6q+1)(6q+2)$$
$$= 6[q(6q+1)(6q+2)]$$

$\boxed{(n-1)\,n\,(n+1) = 6m}$ is divisible by 6. $\because m = q(6q+1)(6q+2)$

• when $n = 6q+2$,

$$(n-1)\,n\,(n+1) = (6q+1)(6q+2)(6q+3)$$
$$= 6[(6q+1)(3q+1)(2q+1)]$$

$\boxed{(n-1)\,n\,(n+1) = 6m}$ is divisible by 6. $\because m = (6q+1)(3q+1)(2q+1)$

• when $n = 6q+3$,

$$(n-1)\,n\,(n+1) = (6q+2)(6q+3)(6q+4)$$
$$= 6[(3q+1)(2q+1)(6q+4)]$$

$\boxed{(n-1)\,n\,(n+1) = 6m}$ is divisible by 6. $\because m = (3q+1)(2q+1)(6q+4)$

• when $n = 6q+4$,

$$(n-1)n(n+1) = (6q+3)(6q+4)(6q+5)$$
$$= 6[(2q+1)(3q+2)(6q+5)]$$

$\boxed{(n-1)n(n+1) = 6m}$ is divisible by 6. $m = (2q+1)(3q+2)(6q+5)$

• when $n = 6q+5$,

$$(n-1)n(n+1) = (6q+4)(6q+5)(6q+6)$$
$$= 6[(6q+4)(6q+5)(q+1)]$$

$\boxed{(n-1)n(n+1) = 6m}$ is divisible by 6. $\because m = (6q+4)(6q+5)(q+1)$

In this way, we can say that (n^3-n) is divisible by 6.

Q.5) Prove that, if a positive integer is of form $(6q+5)$ then it is of the form $(3q+2)$ for some integer q, but not conversely.

Soln:- • Given that, $(6q+5)$ is a positive integer, for some integer q.

• But, we already know that, any positive integer can be of the form $3k$ or $(3k+1)$ or $(3k+2)$.

• Hence, here q may be $3k$ or $(3k+1)$ or $(3k+2)$.

• when $q = 3k$,

$$n = 6q+5$$
$$n = 6(3k)+5$$
$$n = 18k+5 = (18k+3)+2$$
$$n = 3(6k+1)+2$$

$\boxed{n = 3m+2}$, where m is any integer

• when $q = 3k+1$,

$$\Rightarrow n = 6q+5$$
$$= 6(3k+1)+5$$
$$= 18k+6+5$$
$$= (18k+9)+2$$
$$= 3(6k+3)+2$$

$\boxed{n = 3m+2}$ where m is any integer.

• when $q = 3k+2$,

$$\Rightarrow n = 6q+5$$
$$= 6(3k+2)+5$$
$$= (18k+12+5)$$
$$= (18k+15)+2$$
$$= 3(6k+5)+2$$

$\boxed{n = 3m+2}$ where m is any integer

Thus, we can say that, if a positive integer of form $(6q+5)$, then it is of the form $(3q+2)$ for any integer q.

Conversely,

Let us consider, $n = 3q+2$

But we know that, a positive integer can be of the form 6K or $(6k+1)$ or $(6k+2)$ or $(6k+3)$ or $(6k+4)$ or $(6k+5)$.

• when $q = 6k+1$,

$$\Rightarrow n = (3q+2)$$
$$= 3(6k+1)+2$$
$$n = 18k+5$$

$\boxed{n = 6m+5}$ where m is any integer

• when $q = 6k+2$

$\Rightarrow n = 3q+2$

$= 3(6k+2)+2$

$= 18k+6+2$

$= 18k+8$

$= 6(3k+1)+2$

$\boxed{n = 6m+2}$, where 'm' is any integer

But, this is not in the form of $(6q+5)$.

Thus, if n is of the form $(3q+2)$, then it is not necessary that it is of $(6q+5)$.

Q.6) Prove that, square of any positive integer of the form $(5q+1)$ is of the same form.

$\rightarrow$ Let us consider 'n' be any positive integer of the form $(5q+1)$.

$\Rightarrow n = 5q+1$

squaring on both sides,

$n^2 = (5q+1)^2$

$n^2 = (25q^2 + 10q + 1)$

$n^2 = 5(5q^2+2q)+1$

$\Rightarrow \boxed{n^2 = 5m+1}$ where m is some integer.

$\because m = 5q^2+2q$

Thus, the square of any positive integer of the form $(5q+1)$ is of the same form.

Q.7) Prove that, the square of any positive integer is of the form $3m$ or $(3m+1)$ but not of the form $(3m+2)$.

Soln:-

Let us consider, n be any positive integer of the form $3q$ or $(3q+1)$ or $(3q+2)$.

Since, according to Euclid's division lemma for $b=3$:

- when $n=3q$,

$$n^2=(3q)^2$$
$$\Rightarrow n^2=9q^2$$
$$n^2=3(3q^2)$$
$$\boxed{n^2=3m}$$ where 'm' is any integer $\because m=3q^2$

- when $n=3q+1$,

squaring on both sides,

$$\Rightarrow n^2=(3q+1)^2$$
$$\Rightarrow n^2=9q^2+6q+1$$
$$n^2=3(3q^2+2q)+1$$
$$\boxed{n^2=3m+1}$$ where m is any integer $\because m=3q^2+2q$

- when $n=3q+2$

squaring on both sides,

$$n^2=(3q+2)^2$$
$$n^2=9q^2+12q+4=3(3q^2+4q+1)+1$$
$$\boxed{n^2=3m+1}$$, where m is any integer $\because 3q^2+4q+1=m$

Hence, we can say that, the square of any positive integer is of the form $3m$ or $(3m+1)$ but not of the form $(3m+2)$.

q.8) Prove that, the square of any positive integer is of the form $4q$ or $(4q+1)$ for some integer q.

Soln:- Let us consider 'a' be any positive integer.

According to Euclid's division lemma,

$$a = bq + r$$

from given question, $b = 4$

$$a = 4k + r, \quad n \leq r < 4$$

• when $r = 0 \Rightarrow a = 4k$

$$a^2 = 16k^2$$

$$a^2 = 4(4k^2)$$

$$\boxed{a^2 = 4q}$$

$$\Rightarrow q = 4k^2$$

• when $r = 1 \Rightarrow a = 4k+1$

$$a^2 = (4k+1)^2$$

$$= 16k^2 + 1 + 8k$$

$$= 4(4k + 2) + 1$$

$$\boxed{a^2 = 4q+1} \quad \text{where, } q = 4k(4k+2)$$

• when $r = 2 \Rightarrow a = 4k+2$

$$a^2 = (4k+2)^2$$

$$= 16k^2 + 4 + 16k$$

$$= 4(4k^2 + 4k + 1)$$

$$\boxed{a^2 = 4q} \quad \because q = 4k^2 + 4k + 1$$

• when $r = 3 \Rightarrow a = 4k+3$

$$a^2 = (4k+3)^2$$

$$= 16k^2 + 9 + 24k = 4(4k^2 + 6k + 2) + 1$$

$$\boxed{a^2 = 4q+1} \quad \because q = 4k^2 + 6k + 2$$

Hence, the square of any positive integer is either of the form $4q$ or $(4q+1)$, for any integer q.

Q.9.) Prove that, the square of any positive integer is of the form $5q$ or $(5q+1)$, $(5q+4)$ for some integer q.

Soln:- Let us consider, 'a' be any positive integer. According to Euclid's division lemma,

$$a = bq + r$$

• when $b=5$,

$$\Rightarrow a = 5k + r, \quad n \leq r < 5$$

• when $r=1$,

$$\Rightarrow a = 5k+1$$

$$a^2 = (5k+1)^2$$

$$= 25k^2 + 10k + 1$$

$$= 5k(5k+2) + 1$$

$$\boxed{a^2 = 5q+1} \quad \because q = k(5k+2)$$

• when $r=2$,

$$a = 5k+2$$

$$\Rightarrow a^2 = (5k+2)^2$$

$$= 25k^2 + 4 + 20k$$

$$= 5(5k^2 + 4k) + 4$$

$$\boxed{a^2 = 5q+4} \quad \because q = 5k^2 + 4k$$

• when $r=3$,

$$a = 5k+3$$

$$\Rightarrow a^2 = (5k+3)^2$$

$$= 25k^2 + 9 + 30k$$

$$a^2 = 5(5k^2 + 6k + 1) + 4$$

$\boxed{a^2 = 5q+4}$ $\quad \because q = 5k^2+6k+1$

• when $r=4$

$\Rightarrow a = 5k+4$

$a^2 = (5k+4)^2 = 25k^2+20k+16$

$a^2 = 5(5k^2+8k+3)+1$

$\boxed{a^2 = 5q+1}$ $\quad \because q = 5k^2+8k+3$

Hence, the square of any positive integer is of the form $5q$ or $(5q+1)$ or $(5q+4)$, for any integer q.

Q.10) Show that, the square of odd integer is of the form $(8q+1)$, for any integer q.

Soln:- According to Euclid's division lemma;

$a = bq+r \quad , 0 \leq r < b$

when $b=4$,

$\boxed{a = 4q+r}$ $\quad , 0 \leq r < 4$

for $r=4$, $\Rightarrow a = 4q$ which is an even number

for $r=1$ $\Rightarrow a = 4q+1$, which is an odd number

on squaring,

$\Rightarrow a^2 = (4q+1)^2$

$a^2 = 16q^2+1+8q$

$a^2 = 8(2q^2+q)+1$

$\boxed{a^2 = 8m+1}$ $\quad \because m = 2q^2+q$

• when $r=2$

$\Rightarrow a = 4q+2$

$a^2 = (4q+2)^2 \quad a = 2(2q+1)$

$a^2 = 16q^2+16 \quad a = 2(2q+1)$ is an even number

• when, $r=3$

$\Rightarrow a = 4q+3$ is an odd number

on squaring both sides,

$a^2 = (4q+3)^2$

$= 16q^2+9+24q$

$= 8(2q^2+3q+1)+1$

$\boxed{a^2 = 8m+1} \quad \because m = 2q^2+3q+1$

Hence, the square of an odd integer is of the form $(8q+1)$, for any integer q.

Q.11.) Show that any positive odd integer is of the form $(6q+1)$ or $(6q+3)$ or $(6q+5)$, where q is any integer.

Soln:- Let us consider 'a' be any positive integer.

According to Euclid's division lemma,

$a = bq + r, \quad 0 \leq r < b$

put $b=6 \Rightarrow \boxed{a = 6q+r, \quad 0 \leq r < 6}$

• when $r=0 \Rightarrow a = 6q = 2(3q) = 2m$ is an even number

• when $r=1 \Rightarrow a = 6q+1 = 2(3q)+1 = 2m+1$ which is an odd number.

• when $r=2 \Rightarrow a = 6q+2 = 2(3q+1) = 2m$ is an even number.

• when $r=3 \Rightarrow a = 6q+3 = 2(3q+1)+1 = 2m+1$ is an odd number.

• when $r=4 \Rightarrow a = 6q+4 = 2(3q+2)+1$
$= 2m+1$ is an even number

• when $r=5 \Rightarrow a = 6q+5 = 2(3q+2)+1$
$= 2m+1$ is an odd number

Thus, from above all observations it can be seen that any positive odd integer can be of the form $(6q+1)$ or $(6q+3)$ or $(6q+5)$, where q is any integer.

Q.12) Show that, the square of any positive integer cannot be of the form $(6m+2)$ or $(6m+5)$ for any integer m.

Soln:- Let us consider 'a' be positive integer.
According to Euclid's division lemma,

$$a = 6q + r, \quad 0 \le r < 6$$

$$a^2 = (6q+r)^2 = 36q^2 + r^2 + 12qr \quad \because (a+b)^2 = a^2+2ab+b^2$$

$$\Rightarrow \boxed{a^2 = 6(6q^2+2qr)+r^2} \text{ —— ① where } 0 \le r < 6$$

• when $r=0 \Rightarrow a^2 = 6(6q^2) = 6m \quad \because m = 6q^2$ is an integer

• when $r=1 \Rightarrow a^2 = 6(6q^2+2q)+1 = 6m+1$
$\because m = (6q^2+2q)$ is an integer

• when $r=2 \Rightarrow a^2 = 6(6q^2+4q)+4$
$\boxed{a^2 = 6m+4} \quad \because m = 6q^2+4q$ is an integer

• when $r=3$, $a^2 = 6(6q^2+6q)+9$
$= 6(6q^2+6q)+6+3$
$a^2 = 6(6q^2+6q+1)+3$
$\boxed{a^2 = 6m+3}$

• when $r=4$ $\Rightarrow a^2 = 6(6q^2+8q)+16$

$$a^2 = 6(6q^2+8q)+12+4$$

$$a^2 = 6(6q^2+8q+2)+4$$

$\boxed{a^2 = 6m+4}$ $\because m = 6q^2+8q+2$

• when $r=5$ $\Rightarrow a^2 = 6(6q^2+10q)+25$

$$a^2 = 6(6q^2+10q)+24+1$$

$$a^2 = 6(6q^2+10q+4)+1$$

$\boxed{a^2 = 6m+1}$ $\because m = 6q^2+10q+4$

Thus, the square of any positive integer cannot be of the form $(6m+2)$ or $(6m+5)$ for any integer m.

Hence proved.

q.13) Show that, the cube of a positive integer of the form $(6q+r)$, q is an integer & $r = 0,1,2,3,4,5$ is also of the form $(6m+r)$.

Soln:- • Given that, $(6q+r)$ is a positive integer, where q is an any integer and $r = 0,1,2,3,4,5$.

• We know that, any positive integer is of the form $6q$, $(6q+1)$, $(6q+2)$, $(6q+3)$, $(6q+4)$ & $(6q+5)$.

• when $6q$, $(6q)^3 = 216q^3$

$$= 6(36q)^3+0$$

$\boxed{(6q)^3 = 6m+0}$ $\because m = (36q)^3$

• for $(6q+1)$, $(6q+1)^3 = 216q^3+108q^2+18q+1$

$$= 6(36q^3+18q^2+3q)+1$$

$\boxed{(6q+1)^3 = 6m+1}$ $\because m = 36q^3+18q^2+3q$

• for $6q+2$,

$$(6q+2)^3 = 216q^3 + 216q^2 + 72q + 8$$

$$\cancel{= 6(36q^3 + 18q^2 + 3q) + 1}$$

$$\cancel{(6q+2) = 6m+1}$$

$$(6q+2)^3 = 6(36q^3 + 36q^2 + 12q + 1) + 2$$

$$\boxed{(6q+2)^3 = 6m+2} \quad \because m = 36q^3 + 36q^2 + 12q + 1$$

• For $6q+3$,

$$(6q+3)^2 = 216q^3 + 324q^2 + 162q + 27$$

$$= 6(36q^3 + 54q^2 + 27q + 4) + 3$$

$$\boxed{(6q+3)^2 = 6m+3} \quad \because m = 36q^3 + 54q^2 + 27q + 4$$

• For $6q+4$,

$$(6q+4)^3 = 216q^3 + 432q^2 + 288q + 64$$

$$= 6(36q^3 + 72q^2 + 48q + 10) + 4$$

$$\boxed{(6q+4)^3 = 6m+4} \quad \because m = 36q^3 + 72q^2 + 48q + 10$$

• for $6q+5$,

$$(6q+5)^3 = 216q^3 + 540q^2 + 450q + 125$$

$$= 6(36q^3 + 90q^2 + 75q + 20) + 5$$

$$\boxed{(6q+5)^3 = 6m+5} \quad \because m = 36q^3 + 90q^2 + 75q + 20$$

- Thus, the cube of a positive integer of the form $(6q+r)$, q is an integer & $r = 0, 1, 2, 3, 4, 5$ is also of the form $(6m+r)$.

Q.14) Show that one & only one out of $n, (n+4), (n+8), (n+12)$ and $(n+16)$ is divisible by 5, where 'n' is any positive integer.

Soln:- By Euclid's division lemma,

- Let us consider 'n' be any positive integer & $b=5$.
- Let $n = 5q + r$, where q is quotient & r is remainder.

Here, $0 \leq r < 5 \Rightarrow$ remainder may be 0, 1, 2, 3, 4 & 5.

- Thus, we can say that 'n' may be of the form $5q, (5q+1) (5q+2), (5q+3), (5q+4)$.

Case1:

- when $n = 5q$

$\Rightarrow n+4 = 5q+4$

$n+8 = 5q+8$

$n+12 = 5q+12$

$n+16 = 5q+16$

And hence, here n is only divisible by 5.

Case2:

- when $n = 5q+1$

$n+4 = 5q+5 = 5(q+1)$

$n+8 = 5q+9$

$n+12 = 5q+13$

$n+16 = 5q+17$

And hence, here $(n+4)$ is only divisible by 5

Case 3:

- when $n = 5q+2$

$n+4 = 5q+6$

$n+8 = 5q+10 = 5(q+2)$

$n+12 = 5q+14$

$n+16 = 5q+18$

And hence, here $(n+8)$ is only divisible by 5.

Case 4:

- when $n = 5q+3$

$n+4 = 5q+7$

$n+8 = 5q+11$

$n+12 = 5q+15 = (q+3)5$

$n+16 = 5q+19$

And, hence here $(n+12)$ is only divisible by 5.

Case 5:

- when, $n = 5q+4$

$n+4 = 5q+12$

$n+12 = 5q+16$

$n+16 = 5q+20 = 5(q+4)$

And hence, here $(n+16)$ is only divisible by 5.

Thus, here we conclude that one & only one out of n, $(n+4)$, $(n+8)$, $(n+12)$ & $(n+16)$ is divisible by 5.

Hence proved.

Q.15) Show that, the square of an odd integer can be of the form $(6q+1)$ or $(6q+3)$, for some integer q.

→ Let us consider 'a' be an odd integer and $b=6$.

By Euclid's algorithm,

$a = 6m + r$ for any integer $m \geq 0$

And also, $r = 0, 1, 2, 3, 4, 5$ since $0 \leq r < 6$

Thus, we can write

$a = 6m$ or $(6m+1)$ or $(6m+2)$ or $(6m+3)$ or $(6m+4)$ or $(6m+5)$.

Here, we choose $a = (6m+1)$ or $(6m+3)$ or $(6m+5)$ to be an odd integer.

• for $a = 6m+1$,

$$(6m+1)^2 = (36m^2 + 12m + 1)$$
$$= 6(6m^2 + 2m) + 1$$

$\boxed{(6m+1)^2 = 6q+1}$ ∵ q is any integer & $q = 6m^2 + 2m$

• For $a = 6m+3$

$$\Rightarrow (6m+3)^2 = 36m^2 + 36m + 9$$
$$= 6(6m^2 + 6m + 1) + 3$$

$\boxed{(6m+3)^2 = 6q+3}$ where q is any integer & $q = 6m^2 + 6m + 1$

• for $a = 6m+5$,

$$\Rightarrow (6m+5)^2 = 36m^2 + 60m + 25$$
$$= 6(6m^2 + 10m + 4) + 1$$

$\boxed{(6m+5)^2 = 6q+1}$ ∵ q is any integer & $q = 6m^2 + 10m + 4$

Thus, the square of an odd integer of the form $(6q+1)$ or $(6q+3)$ for any integer q.

Hence proved.

Q.16> A positive integer is of the form $(3q+1)$, q being a natural number. Can you write it's square in any form other than $(3m+1)$, $3m$ or $(3m+2)$ for some integer m? Justify your answer.

Soln:- proof:

According to Euclid's lemma,

$$a = bq + r, \quad 0 \le r < b$$

Here, a is any positive integer & $b=3$.

$$\Rightarrow \boxed{a = 3q + r}$$

And here, a be of the form $3q$, $(3q+1)$ or $(3q+2)$.

- For $a = 3q$

$$(3q)^2 = 3(3q^2) = 3m \quad \because m = 3q^2$$

- For $a = 3q+1$

$$\Rightarrow (3q+1)^2 = 9q^2 + 6q + 1 = 3(3q^2+2q)+1$$

$$\boxed{(3q+1)^2 = 3m+1} \quad \because m = 3q^2+2q.$$

- For $a = (3q+2)$,

$$\Rightarrow (3q+2)^2 = 9q^2 + 12q + 4$$

$$= 9q^2 + 12q + 3 + 1$$

$$= 3(3q^2+4q+1)+1$$

$$\boxed{(3q+2)^2 = 3m+1} \quad \because m = 3q^2+4q+1$$

Hence, square of a positive integer of the form $(3q+1)$ is always of the form $(3m+1)$ or $3m$, for any integer m.

Q.17) Show that, the square of any positive integer cannot be of the form $(3m+2)$, where m is a natural number.

Soln:- Let us consider 'a' be any positive integer.

According to Euclid's division lemma,

$$a = bm + r$$

From given that, $b = 3$

$$\Rightarrow \boxed{a = 3m + r} \qquad \because r = 0, 1, 2$$

- when $r = 0 \Rightarrow a = 3m$
- when $r = 1 \Rightarrow a = 3m+1$
- when $r = 2 \Rightarrow a = 3m+2$

Now, • When $a = 3m$

$$\Rightarrow a^2 = (3m)^2 = 9m^2$$

$$a^2 = 3(3m^2) = 3q \qquad \because q = 3m^2$$

- when $a = (3m+1)$

$$\Rightarrow a^2 = (3m+1)^2 = 9m^2 + 6m + 1$$

$$= 3(3m^2 + 2m) + 1$$

$$\boxed{a^2 = 3q+1} \qquad \because q = 3m^2 + 2m$$

- when $a = (3m+2)$

$$\Rightarrow a^2 = (3m+2)^2 = 9m^2 + 12m + 4$$

$$a^2 = 3(3m^2 + 4m + 1) + 1$$

$$\boxed{a^2 = 3q+1} \qquad \because q = 3m^2 + 4m + 1$$

Thus, square of any positive integer cannot be of the form $(3q+2)$, where q is a natural number. Hence proved

Exercise 1.2

Q.1) Define HCF of two positive integers & find the HCF of the following pairs of numbers:

i) 32 & 54

→ Here, we can apply Euclid's division lemma on 54 & 32.

• Thus, $54 = 32 \times 1 + 22$

Since, remainder $\neq 0$

Hence, we can apply Euclid's division lemma on 32 & 22.

• Thus, $32 = 22 \times 1 + 10$

Since, remainder $\neq 0$

Hence, we can apply division lemma on 22 & 10.

• Thus, $22 = 10 \times 2 + 2$

since, remainder $\neq 0$

Hence, we can apply division lemma on 10 & 2.

• Thus, $10 = 2 \times 5 + 0$

Here, remainder $= 0$

Therefore, the HCF of 32 & 54 is 2.

ii) 18 & 24

→ Here, to find HCF we can apply division lemma on 18 & 24.

⇒ $24 = 18 \times 1 + 6$

since here remainder $\neq 0$

Hence, we can apply division lemma on 18 & 6.

⇒ $18 = 6 \times 3 + 0$

Here, remainder $= 0$

Thus, HCF of 18 and 24 is 6.

iii) 70 & 30

→ To find HCF of 70 & 30, we can apply division lemma on 70 & 30.

Thus, $70 = 30 \times 2 + 10$

Here, remainder $\neq 0$

Hence, we can apply division lemma on 30 & 10.

Thus $30 = 10 \times 3 + 0$

Here, remainder $= 0$

Thus, H.C.F of 70 & 30 is 10.

iv) 56 & 88

→ To find H.C.F, we can apply division lemma on 56 & 88.

Thus, $88 = 56 \times 1 + 32$

Since, remainder $\neq 0$

Hence, we can apply division lemma on 56 & 32.

⇒ Thus, $56 = 32 \times 1 + 24$

Here, remainder $\neq 0$

Hence, we can apply division lemma on 32 & 24.

⇒ Thus, $32 = 24 \times 1 + 8$

Here, remainder $\neq 0$

Hence, we can apply division lemma on 24 & 8.

⇒ Thus, $24 = 8 \times 3 + 0$

Thus, the H.C.F. of 56 & 88 is 8.

v) 475 and 495

→ To find H.C.F. we can apply Euclid's division lemma on 495 & 475,

⇒ $495 = 475 \times 1 + 20$

Since, remainder is not zero.

Hence, we can apply division lemma on 475 & 20.

⇒ Thus, $475 = 20 \times 23 + 15$

Since, Here remainder $\neq 0$

Hence, we can apply division lemma on 20 & 15.

⇒ Thus $20 = 15 \times 1 + 5$

Since, Here remainder $\neq 0$

Hence, we can apply division lemma on 15 & 5.

⇒ Thus, $15 = 5 \times 3 + 0$ Here remainder $= 0$

Thus, H.C.F. of 475 and 495 is 5.

vi) 75 and 243

→ To find H.C.F. of 75 and 243, we can apply division lemma on 75 and 243.

⇒ Thus, $243 = 75 \times 3 + 18$

Here, remainder $\neq 0$

Hence, we can apply division lemma on 75 and 18.

⇒ Thus, $75 = 18 \times 4 + 3$

Here, remainder $\neq 0$

Hence, we can apply division lemma on 18 and 3.

⇒ Thus, $18 = 3 \times 6 + 0$, remainder $= 0$

Thus, H.C.F. of 75 & 243 is 3.

vii) ~~155~~ 240 and 6552

→ To find H.C.F. of 242 and 6552, we can apply Euclid's division lemma on 240 & 6552.

Thus, $6552 = 240 \times 27 + 72$

Here, remainder $\neq 0$, we can apply division lemma on 240 & 72.

⇒ Thus, $240 = 72 \times 3 + 24$

Here, remainder $\neq 0$, we can apply division lemma on 72 and 24.

⇒ Thus, $72 = 24 \times 3 + 0$ ∵ remainder $= 0$

Thus, H.C.F. of 240 and 6552 is 24.

viii) 155 and 1385

→ To find H.C.F. of 155 and 1385, we can apply Euclid's division lemma on 155 and 1385.

⇒ Thus $1385 = 155 \times 8 + 145$

Here, remainder $\neq 0$, we can apply division lemma on 155 and 145.

⇒ Thus, $155 = 145 \times 1 + 10$

Since, remainder $\neq 0$, Hence we can apply division lemma on 145 and 10.

⇒ Thus, $10 = 5 \times 2 + 0$.

Hence, HCF of 155 and 1385 is 5.

ix) 100 and 190

→ To find H.C.F. of 100 & 190, we can apply Euclid's division lemma on 100 & 190.

Thus, $190 = 100 \times 1 + 90$

Here, remainder $\neq 0$, we can apply Euclid's division lemma on 100 & 90.

⇒ Thus, $100 = 90 \times 1 + 10$

Since, remainder $\neq 0$, we can apply Euclid's division lemma on 90 & 10.

⇒ $90 = 10 \times 9 + 0$, since remainder $= 0$.

Thus, H.C.F. of 100 and 190 is 10.

x) 105 and 120

→ To find H.C.F. of 105 & 120, we can apply Euclid's division lemma on 105 & 120.

⇒ Thus, $120 = 105 \times 1 + 15$.

Since, here remainder $\neq 0$, we can apply division lemma on 105 & 15.

⇒ Thus, $105 = 15 \times 7 + 0$, since remainder $= 0$

Therefore, H.C.F. of 105 & 120 is 15.

Q.2) Use Euclid's division algorithm to find HCF of

i) 135 and 225

→ Here, the integers given are 135 and 225.

We found that, $225 > 135$.

Hence, we can apply Euclid's division lemma to 225 & 135.

we get, $225 = 135 \times 1 + 90$

Since, remainder $\neq 0$, we can apply division lemma to 135 and 90.

⇒ Thus, $135 = 90 \times 1 + 45$

Here, remainder $\neq 0$, we can apply division lemma to 90 and 45.

⇒ $90 = 45 \times 2 + 0$

Here, remainder $= 0$, Hence the divisor will be the H.C.F.

Thus, the H.C.F. of 225 and 135 is 45.

ii) 196 and 38220

→ Given integers are 196 and 38220 and $38220 > 196$.

According to Euclid's division lemma to 38220 & 196,

⇒ $38220 = 196 \times 195 + 0$

Here, the remainder is zero, then the divisor will be HCF.

⇒ Hence, the HCF of 38220 & 196 is 196.

iii) 867 and 255

→ Here, the given integers are 867 & 255 and $867 > 255$.

By applying Euclid's division lemma to 867 & 255.

$\Rightarrow$ $\underline{867 = 255 \times 3 + 102}$

Since, Here remainder $\neq 0$.

Hence, we can apply division lemma to the divisor 255 and remainder 102.

$\Rightarrow$ Thus, $\underline{255 = 102 \times 2 + 51}$

Here, remainder $\neq 0$, we can apply division lemma on divisor 102 and remainder 51.

$\Rightarrow$ Thus, $\underline{102 = 51 \times 2 + 0}$

Here, remainder $= 0$, then the divisor will be HCF.

Therefore, the HCF of 867 and 255 is 51.

iv) 184, 230 & 276

→ First we can find the HCF of 184 & 230 using Euclid's division lemma.

$\Rightarrow$ $\underline{230 = 184 \times 1 + 46}$ $\because 230 > 184$

Here, remainder $\neq 0$, we can apply division lemma to the divisor 184 and remainder is 46.

$\Rightarrow$ Thus $\underline{184 = 46 \times 4 + 0}$

[illegible]

Here, remainder $= 0$, [illegible]
the divisor will be HCF & hence 46 is HCF of 184 and 230.

Now, we can again use Euclid's division lemma to find HCF of 46 and 276.

$\Rightarrow$ Thus, $276 = 46 \times 6 + 0$

Here, remainder $= 0$,

Thus, the HCF of 276 and 46 is 46.

Therefore, the HCF of 184, 230 & 276 is 46.

v) 136, 170 & 255

Soln:- Here, we can first find the HCF of 136 and 170 by Euclid's division lemma.

$\Rightarrow$ Thus, $170 = 136 \times 1 + 34$

Here, remainder $\neq 0$, so we can apply division lemma to divisor 136 and remainder 34.

$\Rightarrow$ Thus, $136 = 34 \times 4 + 0$

Here, remainder $= 0$, so the divisor will be the HCF.

Thus, here 34 is the HCF of 170 and 136.

Now, we can again use Euclid's division lemma to find HCF of 34 and 255.

$\Rightarrow$ Thus, $255 = 34 \times 7 + 17$

Since, remainder $\neq 0$, so we can apply division lemma on divisor 34 and remainder 17.

$\Rightarrow$ Thus, $34 = 17 \times 2 + 0$.

Here, remainder $= 0$, so the divisor will be the HCF.

Thus, the HCF of 34 & 255 is 17.

Hence, the HCF of 136, 170 and 255 is 17.

Q.3) Find the HCF of the following pair of integers and express it as a linear combination of them.

i) 963 and 657

→ To find HCF we can apply Euclid's division lemma to 963 and 657.

$\Rightarrow 963 = 657 \times 1 + 306$ —— ①

Here, remainder $\neq 0$, so we can apply division lemma on divisor 657 and remainder 306.

$\Rightarrow$ Thus, $657 = 306 \times 2 + 45$ —— ②

Here, remainder $\neq 0$, so we can apply division lemma on divisor 306 and remainder 45.

$\Rightarrow$ Thus, $306 = 45 \times 6 + 36$ —— ③

Here, remainder $\neq 0$, so we can apply division lemma on divisor 45 and remainder 36.

$\Rightarrow$ Thus, $45 = 36 \times 1 + 9$ —— ④

Again, remainder $\neq 0$, so we can apply division lemma on divisor 36 and remainder 9.

$\Rightarrow$ Thus, $36 = 9 \times 4 + 0$ —— ⑤

Thus, we can say that, HCF of 963 & 657 is 9.

Now, to express the HCF as a linear combination of 963 and 657,

we write, $9 = 45 - 36 \times 1$ from eqn ④

$= 45 - [306 - 45 \times 6] \times 1$ from ③

$= 45 - [306 \times 1) + 45 \times 6$

$= 45 \times 7 - 306 \times 1$

$= [657 - 306 \times 2] \times 7 - 306 \times 1$

from ②

$= 657 \times 7 - 306 \times 14 - 306 \times 1$

$= 657 \times 7 - 306 \times 15$

$= 657 \times 7 - [963 - 657 \times 1] \times 15$ from ①

$= 657 \times 7 - 963 \times 15 + 657 \times 15$

$$\boxed{9 = 657 \times 22 - 963 \times 15}$$

ii) 592 and 252.

→ Given numbers are 592 and 252.
To find H.C.F. of 592 and 252, we can use Euclid's division lemma on 592 and 252.

⇒ Thus, $592 = 252 \times 2 + 88$ —— ①

Here, remainder ≠ 0, we can apply division lemma on divisor 252 and remainder 88.

⇒ $252 = 88 \times 2 + 76$ —— ②

As Here, remainder ≠ 0, we can apply division lemma on divisor 88 and remainder 76.

⇒ ~~$76 = 12 \times 6 + 4$~~ $88 = 76 \times 1 + 12$ —— ③

Here, remainder ≠ 0, we can apply division lemma on divisor 76 and remainder 12.

⇒ Thus, $76 = 12 \times 6 + 4$ —— ④

Here, remainder ≠ 0, we can apply division lemma on divisor 12 and remainder 4.

⇒ Thus, $12 = 4 \times 3 + 0$ —— ⑤

Here, remainder = 0

Thus, we conclude that, the HCF of 592 & 252 is 4.

Now, to express the HCF as linear combination of 592 and 252,

we write, $4 = 76 - 12 \times 6$ from eqn (4)

$= 76 - [88 - 76 \times 1] \times 6$ from eqn (3)

$= 76 - 88 \times 6 + 76 \times 6$

$= 76 \times 7 - 88 \times 6$

$= [252 - 88 \times 2] \times 7 - 88 \times 6$ from eqn (2)

$= 252 \times 7 - 88 \times 14 - 88 \times 6$

$= 252 \times 7 - 88 \times 14 - 88 \times 6$

$= 252 \times 7 - 88 \times 20$

$= 252 \times 7 - [592 - 252 \times 2] \times 20$ from eqn (1)

$= 252 \times 7 - 592 \times 20 + 252 \times 40$

$= 252 \times 47 - 592 \times 20$

$$\boxed{4 = 252 \times 47 + 592 \times (-20)}$$

iii) 506 and 1155

→ To find HCF of 506 and 1155, we can apply Euclid's division lemma.

⇒ $1155 = 506 \times 2 + 143$ —— (1)

Here, remainder ≠ 0, we can apply division lemma on divisor 506 and remainder 143.

⇒ Thus, $506 = 143 \times 3 + 77$ —— (2)

Here, remainder ≠ 0, we can apply Euclid's division lemma on divisor 143 & remainder 77.

⇒ Thus, $143 = 77 \times 1 + 66$ —— (3)

Here, remainder ≠ 0, so we can apply Euclid's division lemma on divisor 77 and remainder 66.

⇒ Thus, $77 = 66 \times 1 + 11$ —— ④

Since remainder ≠ 0, we can apply division lemma on divisor 66 and remainder 11.

⇒ $66 = 11 \times 6 + 0$ —— ⑤ ∵ remainder = 0

Thus, we found the HCF of 506 and 1155 as 11.

→ We can write, the HCF 11 as a linear combination of 506 and 1155 as below.

$11 = 77 - 66 \times 1$ from ④

$= 77 - [143 - 77 \times 1] \times 1$ from ③

$= 77 - [143 \times 1 + 77 \times 1]$

$= 77 \times 2 - 143 \times 1$

$= [506 - 143 \times 3] \times 2 - 143 \times 1$ ∵ from ②

$= 506 \times 2 - 143 \times 6 - 143 \times 1$

$= 506 \times 2 - 143 \times 7$

$= 506 \times 2 - [1155 - 506 \times 2] \times 7$ ∵ from ①

$= 506 \times 2 - 1155 \times 7 + 506 \times 14$

$\boxed{11 = 506 \times 16 - 1155 \times 7}$

iv) 1288 and 575

Soln:- According to Euclid's division lemma,

$$1288 = 575 \times 2 + 138 \quad \text{—— ①}$$

Here, remainder is not zero and hence we can apply Euclid's division lemma on divisor 575 and remainder 138.

$$\Rightarrow \quad 575 = 138 \times 4 + 23 \quad \text{—— ②}$$

Since, remainder is not equal to zero and hence we can apply Euclid's division lemma on divisor 138 & remainder 23.

$$\Rightarrow \quad 138 = 23 \times 6 + 0 \quad \text{—— ③}$$

Now, here remainder is zero.

Thus, H.C.F of 1288 and 575 is 23.

Now, we can express HCF of 1288 and 575 as a linear combination of 575 and 1288 as given below.

$$\Rightarrow \quad 23 = 575 - 138 \times 4 \quad \text{from ②}$$

$$= 575 - [1288 - 575 \times 2] \times 4 \quad \text{from ①}$$

$$= 575 - 1288 \times 4 + 575 \times 8$$

$$\boxed{23 = 575 \times 9 - 1288 \times 4}$$

Q.4) Find the largest number which divides 615 and 963, leaving remainder 6 in each case.

Soln:→ Initially, we have to find the required numbers which on dividing does not leave any remainder.

→ for that, we shall substract 6 from both the given numbers.

→ Hence, the required numbers are $615 - 6 = 609$ and $963 - 6 = 957$

Now, we have to find the HCF of 609 and 957.

According to Euclid's division lemma,

$$957 = 609 \times 1 + 348$$

Here, Remainder $\neq 0$

Again we have to apply Euclid's division lemma,

$$609 = 348 \times 1 + 261$$

Here, Remainder $\neq 0$

Again we have to apply Euclid's division lemma,

$$\Rightarrow 348 = 261 \times 1 + 87$$

Here, remainder $\neq 0$

Again we have to apply Euclid's division lemma,

$$261 = 87 \times 3 + 0$$

$\Rightarrow$ HCF of 609 and 957 is 87.

Thus, the required number is 87.

Q.5) If the HCF of 408 and 1032 is expressible in the form $1032m - 408 \times 5$, find m.

Soln:- To find m, first we have to find HCF of 408 and 1032.

By Euclid's division lemma,

$$1032 = 408 \times 2 + 216$$

Here, remainder $\neq 0$, we can apply again Euclid's divison lemma to 408 and 216.

$$\Rightarrow 216 = 192 \times 1 + 24$$

Here, remainder $\neq 0$ so we can apply again Euclid's division lemma to 192 and 24.

$$\Rightarrow 192 = 24 \times 8 + 0$$

Here, remainder $= 0$

$\Rightarrow$ So we can say that the divisor 24 is the HCF of 408 and 1032.

Now, we can write HCF in linear combination as

$$24 = 1032m - 408 \times 5$$

$$1032m = 24 + 408 \times 5$$

$$1032m = 24 + 2040$$

$$1032m = 2064$$

$$\Rightarrow m = 2064/1032$$

$$\boxed{m = 2}$$

Q.6) If the HCF of 657 and 963 is expressible in the form $657x + 963x - 15$, find x.

Soln:- Initially, we will find the HCF of 657 and 963.

By Euclid's divison lemma,

$$963 = 657 \times 1 + 306$$

Here, remainder $\neq 0$, so we can apply Euclid's divison lemma,

$$\Rightarrow 657 = 306 \times 2 + 45$$

Again we can apply Euclid's division lemma,

$$306 = 45 \times 6 + 36$$

As remainder $= 36 \neq 0$, so we can apply division lemma,

$$45 = 36 \times 1 + 9$$

Here, Again remainder $\neq 0$, so we can apply E.D.L.

$$\Rightarrow 36 = 9 \times 4 + 0$$

Now, Here remainder $= 0$

So, we can say that 9 is the HCF of 657 and 963.

Now, we can express HCF in linear combination as follows:

$$9 = 657x + 963(-15)$$

$$9 = 657x - 14445$$

$$9 + 14445 = 657x$$

$$\Rightarrow x = 14454/657$$

$$\therefore \boxed{x = 22}$$

Q.7) An army contigent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Soln:–

Here, Given that,

- An army contigent of 616 members is to march behind an army band of 32 members in a parade.
- And again given that, the two groups are to march in the same number of columns.
- Hence, here we have to find the maximum number of columns in which they are marching
- And for this, we have to find HCF of 616 and 32.

According to Euclid's division lemma,

$$616 = 32 \times 19 + 8$$

$$32 = 8 \times 4 + 0 \qquad \because \text{Remainder} \neq 0$$

Thus, HCF of 616 and 32 is 8

Hence, the maximum number of columns in which army contigent members are marching is 8.

Q.8) A merchant has 120 litres of oil of one kind, 180 litres of another and 240 liters of the third kind. He wants to sell the oil by filling the three kinds of oil in tins of equal capacity, what should be the greatest capacity of such tin?

Sol^n:— Given that,

- The merchant has three different oils of 120 liters, 180 litres and 240 litres respectively.
- So to find the greatest capacity of the tin for filling three different types of oil we have to find simply HCF of the three quantities 120, 180 and 240.

According to Euclid's lemma for 180 and 120,

$$180 = 120 \times 1 + 60$$

Again, $120 = 60 \times 2 + 0$ $\because$ Remainder $\neq 0$

Thus, 60 is the HCF of 180 and 120.

Now, we will find the HCF of 240 and 60.

According to Euclid's division lemma,

$$240 = 60 \times 4 + 0$$

Thus, remainder $= 0$.

Hence, HCF of 240 and 60 is 60.

Thus, we can say that, the tin should be of 60 litres.

Q.9.) During a sale, colour pencils were being sold in packs of 24 each and crayons in packs of 32 each. If you want full packs of both and the same number of pencils and crayons, how many of each would you need to buy?

Soln.:- Here,

- In order to get the full packs of both colour pencils and crayons and also the same numbers in quantity, we have to find the number of packets each need to bought.
- Given that, number of colour pencils in a pack = 24
  and, number of crayons in a pack = 32
- Now, the least number of both pencils and crayons need to purchased is calculated from the LCM only.
- Thus, LCM of 24 and 32 is given by

$$2 \times 2 \times 2 \times 2 \times 2 \times 3 = 96$$

Thus, the number of packs of pencils to be bought $= 96/24$
$= 4$ packs

and also, the number of packs of crayon to be bought
$= 96/32$
$= 3$ packs

Q.10.) 144 cartons of Coke cans and 90 cartons of Pepsi cans are to be stacked in a Canteen. If each stack is of the same height and is to contain cartons of the same drink, what would be the greatest number of cartons each stack would have?

Soln.:-

Given that,

- Number of cartons of coke cans are 144.

And number of cartons of pepsi cans are 90.

- To find the greatest number of cartons in a stack we have to find the HCF of 144 and 90.
- Then, according to Euclid's division lemma,

$$144 = 90 \times 1 + 54$$

Since, remainder $\neq 0 \Rightarrow 90 = 54 \times 1 + 36$

Since, remainder $\neq 0 \Rightarrow 54 = 36 \times 1 + 18$

Since, remainder $\neq 0 \Rightarrow 36 = 18 \times 2 + 0$

As remainder $= 0$, we can say that 18 is the divisor of 144 and 90.

Thus, the greatest number of cartons together in one stack is 18 only.

Q.11) Find the greatest number which divides 285 and 1249 leaving remainders 9 and 7 respe.

Soln:- From given,

The required number divides 285 and 1249 leaving remainder 9 and 7 respectively.

$285 - 9 = 276$ and $1249 - 7 = 1242$

Thus, now we have to find the HCF of 276 and 1242 and which is the required number.

According to Euclid's division lemma,

$$1242 = 276 \times 4 + 138$$

Since, remainder $\neq 0 \Rightarrow 276 = 138 \times 2 + 0$

Thus, HCF of 276 and 1242 is 138.

Thus, the required number is 138.

Q. 12) Find the largest number which exactly divides 280 and 1245 leaving remainders 4 and 3, respectively.

Soln:- Here, given that

We have to find the largest number which exactly divides 280 and ~~14~~ 1245 leaving remainders 4 and 3.

$\Rightarrow 280-4 = 276$ and $1245-3 = 1242$

Thus, the required numbers are 276 and 1242.

We have to find the HCF of 276 and 1242.

According to Euclid's division lemma,

$1242 = 276 \times 4 + 138$

Since, remainder $\neq 0$ $276 = 138 \times 2 + 0$

Thus, the HCF of 280 and 1245 is 138 only.

Hence, the required number is 138.

Q. 13) What is the largest number which that divides 626, 3127 and 15628 and leaves remainder of 1, 2 and 3 respectively.

Soln:- From given condition, we can write

$626-1 = 625$

$3127-2 = 3125$

and $15628-3 = 15625$

This numbers has to be exactly divisible by the number.

And hence, required number is the HCF of 625, 3125 and 15625.

Now, we can find the HCF of 625 and 3125.

By Euclid's division lemma,

$3125 = 625 \times 5 + 0$

Thus, HCF of 3125 and 625 is 625.

Now, we can find HCF of 625 and 15625.

According to Euclid's division lemma,

$15625 = 625 \times 25 + 0$

Thus, the HCF of 625 and 15625 is 625.

Thus, HCF of 625, 3125 and 15625 is 625 only.

Hence, the required number is 625.

Q.14.) Find the greatest number that will divide 445, 572 and 699 leaving remainders 4, 5 and 6 respe.

Soln:- From given example,

we can write, $445 - 4 = 441$, $572 - 5 = 567$ and $699 - 6 = 693$ } This numbers has to be divisible by the number.

Hence, we can find the required number by finding the HCF of 441, 567 and 693.

Now, we can find HCF of 441 and 567.

According to Euclid's division lemma,

$\Rightarrow 567 = 441 \times 1 + 126$ ∵ Remainder $\neq 0$

$441 = 126 \times 3 + 63$ ∵ Remainder $\neq 0$

$126 = 63 \times 2 + 0$

Since, Remainder $= 0 \Rightarrow$ HCF $(441, 567) = 63$

Now, we can find the HCF of 63 and 693 by Euclid's division lemma,

$693 = 63 \times 11 + 0$

Thus, HCF of 68 and 693 is found to be 63.

Hence, the HCF of $(441, 567, 693) = 63$.

Hence, the required number is 63.

Q.15) find the greatest number which divides 2011 and 2623 leaving remainders 9 and 5 respe.

Soln:- From given condition, we can write

2011 − 9 = 2002 and 2623 − 5 = 2618 are divisible exactly by the number.

And hence, the required number is the HCF of 2002 & 2618.

According to Euclid's division lemma,

$2618 = 2002 \times 1 + 616$ ∵ Remainder ≠ 0

$2002 = 616 \times 3 + 154$ ∵ Remainder ≠ 0

$616 = 154 \times 4 + 0$

Here, remainder = 0, thus the HCF of 2002 & 2618 is 154.

Hence, the required number is 154.

Q.16.) Using Euclid's division algorithm, find the largest number that divides 1251, 9377 and 15628 leaving remainders 1, 2 and 3 respe.

Soln:- From given condition, we can write

1251 − 1 = 1250, 9377 − 2 = 9375 and 15628 − 3 = 15625 has to be exactly divisible by the number.

And hence, the required number is HCF of 1250, 9375 and 15625.

Now, we have to find HCF of 1250 and 9375,

According to Euclid's division lemma,

$9375 = 1250 \times 7 + 625$ ∵ remainder ≠ 0

$1250 = 625 \times 2 + 0$

Here, remainder = 0 ⇒ 625 is the HCF of 1250 & 9375.

Now, we have to find HCF of 625 and 15625.

According to Euclid's division lemma,

$15625 = 625 \times 25 + 0$

Thus, HCF of 625 and 15625 is found to be 625.

Thus, HCF (1250, 9375, 15625) = 625

Thus, the required number is 625.

Q.17) Two brands of chocolates are available in packs of 24 & 15 respectively. If I need to buy an equal number of chocolates of both kinds, what is the number of boxes of each kind I would need to buy?

Soln:- Given condition is,

Number of chocolates ~~of~~ 1st brand in a pack = 24

And Number of chocolates ~~of~~ 2nd brand in a pack = 15

Thus, the least number of both brands of chocolates need to be purchased is given by the LCM of 24 and 15.

Thus, the LCM of 24 and 15 = $2 \times 2 \times 2 \times 3 \times 5 = 120$

Hence, the total number of packets of 1st brand to be bought $= \frac{120}{24} = 5$

And, the number of packets of 2nd brand to be bought

$= \frac{120}{15} = 8$

Q.18) A mason has to fit a bathroom with square marble tiles of the largest possible size. The size of the bathroom is 10ft. by 8ft. What would be the size in inches of the tile required that has to be cut and how many such tiles are required?

Soln:-

From given condition,

we can find the size of bathroom,

size of bathroom = 10ft. × 8ft.

$= (10 \times 12)$ inch by (8×12) inch

$= 120$ inch by 96 inch

$\because$ 1ft = 12 inch

Now, the tile having largest size is nothing but the ~~LCM~~ HCF of 120 and 96.

We can find HCF of 120 & 96 by Euclid's division lemma,

$120 = 96 \times 1 + 24$ $\because$ Remainder $\neq 0$

$96 = 24 \times 4 + 0$

Thus, the HCF of 120 and 96 is found to be 24.

Hence, the largest size of tile which we required is 24 inches.

Thus, the number of tiles required $=$ (area of bathroom)/(area of tile)

$= (120 \times 96)/(24 \times 24)$

$= 5 \times 4$

$= 20$ tiles

Hence, we can say that, the 20 tiles each having size 24 inch by 24 inch are required to be cut.

Q.19) 15 pastries and 12 biscuit packets have been donated for a school fete. These are to be packed in several smaller identical boxes with the same number of pastries and biscuits packets in each. How many biscuit packets and how many pastries will each box contain?

Soln:-

From given condition, we can write

Number of pastiet pastries = 15

And Number of biscuit packets = 12

Thus, the required number of boxes which are having equal number of pastries as well as biscuits will be equal to the HCF of 15 and 12.

Thus, by applying Euclid's division lemma,

we have, $15 = 12 \times 1 + 3$

$12 = 3 \times 4 + 0$

Thus, HCF of 15 and 12 is 3.

Thus, the number of boxes required are 3 only.

Hence, each box will have $\frac{15}{3} = 5$ pastries.

And $12/3 = 4$ biscuits packs also.

Q.20.) 105 goats, 140 donkeys and 175 cows have to be taken across a river. There is only one boat which will have to make many trips in order to do so. The lazy boatman has his own conditions for transporting them. He insists that he will take the same number of animals in every trip and they have to be of the same kind. He will naturally like to take the largest possible number each time. Can you tell how many animals went on each trip?

Solⁿ:- From given condition,

Number of goats are 105,

Number of donkeys are 140

And Number of cows are 175.

Thus, The HCF of 105, 140 and 175 gives the largest number of animals in one trip.

We can find HCF of 105 and 140 by Euclid's division lemma,

$140 = 105 \times 1 + 35$ ∵ Remainder $\neq 0$

$105 = 35 \times 3 + 0$

Thus, the HCF of 140 and 105 is 35.

Now, we can find the HCF of ~~to~~ 35 and 175 by Euclid's division lemma,

$175 = 35 \times 5 + 0$

Thus, the HCF of $(105, 140, 175) = 35$.

Hence, we can say that 35 animals went on each trip.

Q.21.) The length, breadth and height of a room are 8m 25cm, 6m 75cm and 4m 50cm respectively. Determine the largest longest rod which can measure the three diemensions of the room exactly.

Soln:- From given question, we can write

length of the room = 8m 25cm = 825cm

breadth of the room = 6m 75cm = 450 cm

Hence, the requred longest rod which is used to measure the room exactly is nothing but the HCF of 825, 675 & 450.

We find HCF of 675 and 450 by Euclid's division lemma,

$675 = 450 \times 1 + 225$ ∵ Remainder $\neq 0$

$\Rightarrow 450 = 225 \times 2 + 0$

Thus, we found HCF of 675 & 450 is 225.

The HCF of 225 and 825 by Euclid's division lemma is

$825 = 225 \times 3 + 150$

$\Rightarrow 225 = 150 \times 1 + 75$

$150 = 75 \times 2 + 0$

Remainder $= 0$, Hence, HCF of 225 and 825 is 75.

Thus, the HCF of 825, 675 and 450 is 75.

Thus, we can say that, the length of the longest rod is 75cm or 0.75m.

Q.22) Express the HCF of 468 and 222 as $468x + 222y$, where x, y are integers in two different ways.

Soln:- From given condition, we can write

HCF of 468 and 222 as,

$$HCF(468, 222) = 468x + 222y$$

where, x & y are integers in two different ways.

Now, here integers are the 468 & 222 and also

$$468 > 222.$$

According Euclid's division lemma, we get

$$468 = 222 \times 2 + 24 \quad \text{——} \; (1)$$

Again, remainder $\neq 0$

$$\Rightarrow \quad 222 = 24 \times 9 + 6 \quad \text{——} \; (2)$$

Again, remainder $\neq 0$

$$24 = 6 \times 4 + 0 \quad \text{——} \; (3)$$

Remainder $= 0 \Rightarrow$ Thus, 6 is the ~~divi~~ HCF of 468 & 222.

Now, to express HCF as a linear combination of 468 and 222, we can write

$$6 = 222 - 24 \times 9 \quad \because \text{from } (2)$$

$$= 222 - (468 - 222 \times 2) \times 9 \quad \because \text{from } (1)$$

$$= 222 - 468 \times 9 + 222 \times 18$$

$6 = 222 \times 19 - 468 \times 9$

$= 468(-9) + 222(19)$

$\therefore \boxed{6 = 468x + 22y}$

where $\boxed{x = -9 \text{ and } y = 19}$

Exercise 1.3

Q.1) Express each of the following integers as a product of it's prime.

i) 420

→ To express 420 as a product of it's prime we solve,

2	420
2	210
5	105
7	21
3	3
	1

$420 = 2 \times 2 \times 3 \times 5 \times 7$

$\boxed{\therefore 420 = 2^2 \times 3 \times 5 \times 7}$

ii) 468

2	468
2	234
3	117
3	39
13	13
	1

$468 = 2 \times 2 \times 3 \times 3 \times 13$

$\boxed{\therefore 468 = 2^2 \times 3^2 \times 13}$

iii) 945

5	945
3	189
3	63
3	21
7	7
	1

$945 = 3 \times 3 \times 3 \times 5 \times 7$

$\boxed{\therefore 945 = 3^2 \times 5 \times 7}$

iv) 7325

→

5	7325
5	1465
293	293
	1

$7325 = 5 \times 5 \times 293$

$\boxed{\therefore 7325 = 5^2 \times 293}$

Q.2) Determine the prime factorization of each of the following positive integer:

i) 20570

→

5	20570
2	4114
11	2057
11	187
17	17
	1

Thus, $20570 = 2 \times 5 \times 11 \times 11 \times 17$

$\boxed{\therefore 20570 = 2 \times 5 \times 11^2 \times 17}$

ii) 58500

5	58500
5	11700
5	2340
2	468
2	234
3	117
3	39
13	13
	1

Thus, $58500 = 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 5 \times 13$

$\boxed{\therefore 58500 = 2^2 \times 3^2 \times 5^3 \times 13}$

iii) 45470971

7	45470971
7	6495853
13	927979
13	71383
17	5491
17	323
19	19
	1

Thus, $45470971 = 7 \times 7 \times 13 \times 13 \times 17 \times 17 \times 19$

$$\boxed{\therefore 45470971 = 7^2 \times 13^2 \times 17^2 \times 19}$$

Q.3. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

→ We know that, there are two types of numbers they are prime numbers and composite numbers.

Prime numbers are those which are having factors 1 and the number itself only.

Thus,
$$7 \times 11 \times 13 + 13 = 13 \times (7 \times 11 + 1)$$
$$= 13 \times (77 + 1)$$
$$= 13 \times 78$$
$$= 13 \times 13 \times 6$$

Thus, the given expression has 6 and 13 are it's factors.

And hence, we can say that it is a composite number.

Q.4) Check whether

Similarly, we can write

$$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = 5 \times (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1)$$

$$= 5 \times (1008 + 1)$$

$$= 5 \times 1009$$

Thus, here 1009 is a prime number, the given expression has 5 and 1009 as it's factors other than 1 and the number itself. Hence, we can say it is also a composite number.

Q.4.) Check whether 6^n can end with the digit 0 for any natural number n.

→ • We can find the factor of 6 here, to check whether 6^n can end with digit 0 for any natural number n.

• And the factors of 6 are 2 and 3.

So, $6^n = (2 \times 3)^n$

$6^n = 2^n \times 3^n$

Here, the prime factorization of 6 does not contains 5 and 2 as it's factor together.

Thus, we can say that 6^n can never end with the digit 0 for any natural number n.

Q. 5.) Explain why $3 \times 5 \times 7 + 7$ is a composite number.

→ We know that, there are two types of numbers.

Prime Numbers: The numbers which are divisible by the number itself and 1 only are called as prime numbers.

e.g. $3 = 3 \times 1$, $5 = 5 \times 1$

Composite Numbers: The numbers which are having factors other than 1 and the number itself are called as composite number.

e.g. $4 = 2 \times 2 = 4 \times 1$

Here, $3 \times 5 \times 7 + 7 = 7 \times (3 \times 5 + 1)$

$= 7 \times (15 + 1)$

$\boxed{3 \times 5 \times 7 + 7 = 7 \times 16}$

Here, the given equation has factors 7 and 16 and hence we can say that it is a composite number.

Exercise 1.4

Q.1) Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of integers.

i) 26 and 91

→ The integers given are 26 and 91.

First we find prime factors of 26 and 91.

$26 = 2 \times 13$

$91 = 7 \times 13$

$\therefore$ LCM $(26, 91) = 2 \times 7 \times 13 = 182$

And HCF $(26, 91) = 13$

Now, we can verify the relation,

LCM × HCF $= 182 \times 13 = 2366$

& Product of integers $= 26 \times 91 = 2366$.

Thus, Here LCM × HCF = product of integers = 2366

Hence proved.

ii) 510 and 92

→ The given integers are 510 and 92.

First here, we can find prime factors of 510 and 92.

$510 = 2 \times 3 \times 5 \times 17$

$92 = 2 \times 2 \times 23$

$\therefore$ LCM $(510, 92) = 2 \times 2 \times 3 \times 5 \times 23 \times 17 = 23460$

And HCF $(510, 92) = 2$

Now, we can verify the given relation,

LCM × HCF $= 23460 \times 2 = 46920$

product of integers $= 510 \times 92 = 46920$

Thus, here LCM × HCF = product of integers = 46920

Hence proved.

iii) 336 and 54

→ Here, given integers are 336 and 54.

Now, initialy we find prime factors of 336 and 54.

$336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7$

$54 = 2 \times 3 \times 3 \times 3$

$\therefore$ LCM (336, 54) $= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times \times 3 \times 7 = 3024$

And HCF (336, 54) $= 2 \times 3 = 6$

Now, we verify the given relation,

LCM $\times$ HCF $= 3024 \times 6 = 18144$

And product of integers $= 336 \times 54 = 18144$

$\therefore$ LCM $\times$ HCF = product of integers.

Hence proved.

Q.2.) Find the LCM and HCF of the following integers by applying the prime factorisation method.

i) 12, 15 and 21

→ Given integers are 12, 15 and 21.

Initialy, we find the prime factors of integers 12, 15 and 21.

Thus, $12 = 2 \times 2 \times 3$

$15 = 3 \times 5$

$21 = 3 \times 7$

Here, LCM (12, 15, 21) $= 2 \times 2 \times 3 \times 5 \times 7 = 420$

And HCF (12, 15 & 21) $= 3$

ii) 17, 23 and 29

→ Here given integers are 17, 23 & 29.

Initialy, we find prime factors of integers 17, 23 and 29.

$17 = 1 \times 17$

$23 = 1 \times 23$

$29 = 1 \times 29$

Thus, LCM $(17, 23, 29) = 1 \times 17 \times 23 \times 29 = 11339$

And HCF (17, 23 and 29) = 1

iii) 8, 9 and 25

→ Here, given integers are 8, 9 and 25.

Initialy we find the prime factors of 8, 9 and 25.

Thus, $8 = 2 \times 2 \times 2$

$9 = 3 \times 3$

$25 = 5 \times 5$

Hence, LCM $(8, 9, 25) = 2^3 \times 3^2 \times 5^2 = 1800$

And hence HCF (8, 9, 25) = 1

iv) 40, 36 and 126

→ Initialy we find the prime factors of 40, 36 and 126.

Thus, $40 = 2 \times 2 \times 2 \times 5$

$36 = 2 \times 2 \times 3 \times 3$

$126 = 2 \times 3 \times 3 \times 7$

So, LCM of 40, 36 & 126 $= 2^3 \times 3^2 \times 5 \times 7$

$\therefore$ LCM (40, 36, 126) = 2520

And HCF (40, 36, 126) = 2

v) 84, 90 and 120

→ Initialy, we find the prime factors of 84, 90 & 120.

So, $84 = 2 \times 2 \times 3 \times 7$

$90 = 2 \times 3 \times 3 \times 5$

$120 = 2 \times 2 \times 2 \times 3 \times 5$

Hence, LCM $(84, 90, 120) = 2^3 \times 3^2 \times 5 \times 7$

LCM (84, 90, 120) = 2520

And HCF (84, 90, 120) = 6

vi) 24, 15 and 36

→ Initialy we find the prime factors of integers 24, 15 and 36.

so $24 = 2\times2\times2\times3$

$15 = 3\times5$

$36 = 2\times2\times3\times3$

Hence, LCM $(24, 15, 36) = 2\times2\times2\times3\times3\times5$

$= 2^3\times3^2\times5$

$\therefore$ LCM $(24, 15, 36) = 360$

And HCF $(24, 15, 36) = 3$

Q.3. Given that HCF (306, 657) = 9, find LCM (306, 657).

→ Given integers are 306 and 657.

we allready know that,

LCM × HCF = product of two integers

⇒ LCM = product of two integers / HCF

$= (306\times657)/9$

LCM = 22338

Thus, LCM (306, 657) = 2238

Q.4. Can two numbers have 16 as their HCF and 380 as their LCM? Give reason.

Solⁿ → First we divide 380 by 16.

```
     23
16 | 380
     32
   -----
     60
     48
   -----
     12
```

After dividing 380 by 16 we get 23 as a quotient and 12 as a remainder.

Now, here LCM is not strictly or exactly divisible by the HCF.

Hence, its can be said that two numbers cannot have 16 as their HCF and 380 as their LCM.

Q.5 The HCF of two numbers is 145 and their LCM is 2175. If one number is 725, find the other.

Soln → Given that,

The HCF of two numbers is 145.

And the LCM of same two numbers is 2175.

And, one of the number from them is 725.

we have,

LCM × HCF = product of numbers

LCM × HCF = first number × second number

2175 × 145 = 725 × second number

Thus, second number $= \frac{2175 \times 145}{725} = 435$.

Hence, the other number is found to be 435.

Q.6. The HCF of two numbers is 16 and their product is 3072. Find their LCM.

Soln → Here, Given that

The two numbers are having HCF = 16

And product of two numbers = 3072

We already know that,

LCM × HCF = product of two numbers

LCM × 16 = 3072

$LCM = \frac{3072}{16} = 192$

Thus, the LCM of two numbers is 192.

Q.7. The LCM and HCF of two numbers are 180 and 6 respe. If one of the numbers is 30 find the other number.

Solⁿ→ Here, Given that

The LCM and HCF of two numbers are 180 & 6 respe.

And one of the number is 30.

we have,

$$LCM \times HCF = \text{first number} \times \text{second number}$$

$$180 \times 6 = 30 \times \text{second number}$$

$$\Rightarrow \text{second number} = (180 \times 6)/30 = 36.$$

Hence, the second number is found to be 36.

Q.8. Find the smallest number which when increased by 17 is exactly divisible by both 520 and 468.

Solⁿ→ Now, initially we find the smallest number which is exactly divisible by both 520 & 468.

That means, we have to find the LCM of 520 & 468.

By prime factorization method,

we write $520 = 2^3 \times 5 \times 13$

$468 = 2^2 \times 3^2 \times 13$

Hence, $LCM(520, 468) = 2^3 \times 3^2 \times 5 \times 13 = 4680$

Thus, we can say that 4680 is the smallest number which is divisible by both 520 and 468 and hence remainder will be zero.

But, here we have to find the smallest number which when increased by 17 is exactly divisible by 520 & 468.

Thus, $4680 - 17 = 4663$

∴ 4663 is the smallest number which when increased by 17 is exactly divisible by both 520 and 468.

Q.9. Find the smallest number which leaves remainders 8 and 12 when divisible by 28 and 32 respe.

Soln → Initially, we have to find the smallest number which is exactly divisible by 28 and 32.

That means, we have to find the LCM of two numbers.

Then, by prime factorization method

$$28 = 2 \times 2 \times 7$$

$$32 = 2^5$$

$$\therefore LCM(28, 32) = 2^5 \times 7 = 224$$

Now, 224 is the smallest number which is exactly divisible by 28 and 32 and hence remainder is zero.

But, here we have to find the number which gives remainder 8 and 12 when divided by 28 and 32 respe.

Thus, it is found by

$$224 - 8 - 12 = 204$$

Hence, we can say that 204 is the smallest number which gives remainder 8 and 12 when divided by 28 and 32 respectively.

Q.10. What is the smallest number that, when divided by 35, 56 and 91 leaves remainders of 7 in each case.

Soln:→ Here, first we have to find the smallest number which is exactly divisible by all 35, 56 and 91.

That means, we have to find LCM of 35, 56 and 91.

By prime factorisation method,

$$35 = 5 \times 7$$

$$56 = 2^3 \times 7$$

$$91 = 13 \times 7$$

Thus, LCM $(35, 56, 91) = 2^3 \times 7 \times 5 \times 13 = 3640$

Hence, we can say that, the smallest number which is divided by 35, 56 & 91 leaves remainder of 7 in each case. is 3640.

And it can be found by,

$$3640 + 7 = 3647.$$

Hence, 3647 is the smallest number which leaves remainder 7 in each case when divided by 35, 56 and 91.

Q.11) A rectangular courtyard is 18m 72cm long and 13m 20cm broad. It is to be paved with square tiles of the same size. Find the least possible number of such tiles.

Soln → Here, given that

Length of courtyard = 18m 72cm = 1800cm + 72cm = 1872cm

And breadth of courtyard = 13m 20cm = 1300cm + 20cm = 1320cm

And here, the size of of the tile which is to be paved on the rectangular yard is equal to the HCF of length and breadth for the given rectangular yard.

To find HCF, we factorise the numbers 1872 and 1320.

Thus, $1872 = 2^4 \times 3^2 \times 13$

$1320 = 2^3 \times 3 \times 5 \times 11$

Thus, HCF $(1872, 1320) = 2^3 \times 3 = 24$

Hence, we can say that the length of the side of the square shaped tile is 24 cm.

Hence, the number of tiles needed = $\dfrac{(\text{area of courtyard})}{(\text{area of a square tile})}$

Here, Area of courtyard = length × breadth

$= (1872 \times 1320)\ cm^2$

And area of a square tile = $(\text{side})^2 = (24cm)^2$

Hence, the number of tiles required $= \frac{(1872 \times 1320)}{(24)^2}$

∴ The number of tiles required $= 4290$

Hence, here least possible number of tiles required are 4290.

Q.12) Find the greatest number of 6 digits exactly divisible by 24, 15 and 36.

Soln → Here we already know that,

The greatest number which is 6 digit number is 999999.

Now, we take an assumption that,

999999 is divisible by 24, 15 and 36 exactly.

Then, we can say that

The LCM (24, 15, 36) should be ~~divisible~~ b divide 999999 exactly.

Now, by prime factorisation,

$24 = 2 \times 2 \times 2 \times 3$

$15 = 3 \times 5$

$36 = 2 \times 2 \times 3 \times 3$

Thus, LCM $(24, 15, 36) = 360$

Here, $\frac{(999999)}{360} = 2777 \times 360 + 279$

And remainder is 279

Hence, we can say that, the greatest number which is divisible by all the three number should be

$999999 - 279 = 999720$

Thus, 999720 is the greatest 6 digit number which is exactly divisible by 24, 15 and 36.

Q.13) Determine the number nearest to 110000 but greater 100000 which is exactly divisible by each of 8, 15 and 21.

Solⁿ → • Now, first we find the LCM of 8, 15 and 21 here.

By prime factorisation method,

$8 = 2 \times 2 \times 2$

$15 = 3 \times 5$

$21 = 3 \times 7$

Thus, $LCM(8, 15, 21) = 2^3 \times 3 \times 5 \times 7 = 840$

Thus, when 110000 is divided by 840, the remainder is 800.

• Thus, $110000 - 800 = 109200$ is divisible by each of 8, 15 and 21.

Also, we can write, $110000 + 40 = 110040$ is also divisible by each of 8, 15 and 21.

• Hence, 109200 and 110040 both are greater than 100000 but 110040 is greater than 110000.

• In this way, 109200 is the nearest number nearest to 110000 and greater than 100000 which is exactly divisible by each of 8, 15 and 21.

Q.14) Find the least number that is divisible by all the numbers between 1 and 10 (both inclusive).

Solⁿ → From the given condition,

Here, the LCM of numbers between 1 and 10 will be the number which is least and divisible by all the numbers between 1 and 10.

So we can find prime factor as follows:

$1 = 1$ $\quad$ $6 = 2 \times 3$

$2 = 2$ $\quad$ $7 = 7$

$3 = 3$ $\quad$ $8 = 2 \times 2 \times 2$

$4 = 2 \times 2$ $\quad$ $9 = 3 \times 3$

$5 = 5$ $\quad$ $10 = 2 \times 5$

Thus, Here LCM is $= 2^3 \times 3^2 \times 5 \times 7 = 2520$

Hence, the least number which is divisible by all the numbers between 1 and 10 (both inclusive) is 2520.

Q.15) A circular field has a circumference of 360 km. Three cyclists starts together and can cycle 48, 60 and 72 km a day, round the field. When they meet again?

Soln → Here, to find the time taken by cyclists before they meet again we have to find the individual time required to complete the total distance by each cyclist.

Now,

$$\left.\begin{array}{c}\text{the number of days required} \\ \text{by cyclist to cover the} \\ \text{total circular distance}\end{array}\right\} = \frac{\text{(Total distance of circular path)}}{\text{(distance covered in 1 day by a cyclist)}}$$

Now, 1st cyclist: Total days $= \frac{360}{48} = 7.5 = 180$ hrs

2nd cyclist: total days $= \frac{360}{60} = 6 = 144$ hrs.

3rd cyclist: total days $= \frac{360}{72} = 5 = 120$ hrs

Now, we can find the LCM of 180, 144 and 120 which gives the time after which all the three cyclists meets again.

By prime factorization method,

$180 = 2^2 \times 3^2 \times 5$

$144 = 2^4 \times 3^2$

$120 = 2^3 \times 3 \times 5$

Thus, here LCM (180, 144, 120) $= 2^4 \times 3^2 \times 5 = 720$

Hence, we can say that all the three cyclists meets at a point again after 720 hrs.

Thus, 720 hrs $= \frac{720}{24}$ days $= 30$ days

In this way, we can say that three cyclists will meet again after 30 days.

Q.16.) In a morning walk three persons step off together, their steps measure 80 cm, 85 cm and 90 cm respectively. What is the minimum distance each should walk so that he can cover the distance in complete steps?

Soln → Now, here the distance covered in complete steps is the LCM of the measures of their steps i.e. 80 cm, 85 cm and 90 cm.

By prime factorization method,

$80 = 2^4 \times 5$

$85 = 17 \times 5$

$90 = 2 \times 3 \times 3 \times 5$

Thus, LCM (80, 85, 90) $= 2^4 \times 3^2 \times 5 \times 17 = 12240$ cm

$= 122$ m 40 cm

Thus, 122 m 40 cm is the required minimum distance that each should walk so that all can cover the same distance in a complete step.

By prime factorization method,

$180 = 2^2 \times 3^2 \times 5$

$144 = 2^4 \times 3^2$

$120 = 2^3 \times 3 \times 5$

Thus, here $LCM(180, 144, 120) = 2^4 \times 3^2 \times 5 = 720$

Hence, we can say that all the three cyclists meets at a point again after 720 hrs.

Thus, $720 \text{ hrs} = \frac{720}{24} \text{ days} = 30 \text{ days}$

In this way, we can say that three cyclists will meet again after 30 days.

Q.16.) In a morning walk three persons step off together, their steps measure 80cm, 85cm and 90 cm respectively. What is the minimum distance each should walk so that he can cover the distance in complete steps?

Soln → Now, here the distance covered in complete steps is the LCM of the measures of their steps i.e. 80cm, 85cm and 90cm.

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$80 = 2^4 \times 5$

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$90 = 2 \times 3 \times 3 \times 5$

Thus, $LCM(80, 85, 90) = 2^4 \times 3^2 \times 5 \times 17 = 12240 \text{ cm}$

$= 122 \text{ m } 40 \text{ cm}$

Thus, 122m 40cm is the required minimum distance that each should walk so that all can cover the same distance in a complete step.

Exercise 1.5

Q.1.) Show that the ~~following~~ numbers are irrational.

Soln:→ i) $1/\sqrt{2}$

Here, to prove $1/\sqrt{2}$ is a irrational number, we use contradiction method.

Let us assume that, $1/\sqrt{2}$ is a rational number.

So, we can write, $\therefore \frac{1}{\sqrt{2}} = r$

$$\frac{1}{r} = \frac{1}{\sqrt{2}}$$

As 'r' is a rational number then $\frac{1}{r} = \sqrt{2}$ is also a rational number.

But, we allready know that, $\sqrt{2}$ is a irrational number.

Hence, our supposition is wrong.

Thus, $1/\sqrt{2}$ is said to be irrational number.

ii) $7\sqrt{5}$

→ Here, to prove $7\sqrt{5}$ is a irrational number, we use contradiction method.

We consider as $7\sqrt{5}$ is a rational number.

Thus, $7\sqrt{5}$ can be expressed as $7\sqrt{5} = \frac{a}{b}$

where a, b are integers

$\Rightarrow 7\sqrt{5} = a/b$

$\Rightarrow \sqrt{5} = a/7b$

Since a, 7, b are integers

So $a/7b$ is a rational number.

$\Rightarrow \sqrt{5}$ is a rational number

But, it is contradiction to our supposition.

Hence, we can say that our assumption is wrong.

Hence, $7\sqrt{5}$ is a irrational number

Hence proved.

iii) $6+\sqrt{2}$

→ To prove $(6+\sqrt{2})$ is a irrational number, we use contradiction method.

Let us consider $(6+\sqrt{2})$ is a rational number.

Thus, we can write $(6+\sqrt{2})$ as

$6+\sqrt{2} = a/b$, where a & b are integers

$\Rightarrow \sqrt{2} = a/b - 6$

$\Rightarrow \sqrt{2} = (a-6b)/b$

Since, a & b are integers

So $(a-6b)/b$ is also a rational number.

$\Rightarrow \sqrt{2}$ is a rational number.

But, it is contradiction to our assumption.

Hence, our assumption is wrong.

Thus, $(6+\sqrt{2})$ is a irrational number

iv) $(3-\sqrt{5})$

→ Here, to prove $(3-\sqrt{5})$ is a irrational number we use contradiction method.

Let us assume $(3-\sqrt{5})$ is a rational number.

So, $(3-\sqrt{5})$ can be expressed as,

$(3-\sqrt{5}) = a/b$

$\Rightarrow \sqrt{5} = a/b + 3$

$\Rightarrow \sqrt{5} = (a+3b)/b$

Since a & b are integers

So $(a+3b)/b$ are also integers. $\Rightarrow (a+3b)/b$ is a rational number

Hence, $\sqrt{5}$ is also a rational number.

But, it is contradiction to our assumption.

Hence, our assumption is wrong.

Thus, $(3-\sqrt{5})$ is a irrational number.

Hence proved.

Q.2) Prove that the following numbers are irrationals.

i) $2/\sqrt{7}$

→ Here, to prove $2/\sqrt{7}$ is a irrational number we use contradiction method.

Let us suppose, $2/\sqrt{7}$ is a rational number.

So it can be expressed as $\frac{2}{\sqrt{7}} = a/b$

where a & b are integers.

$\Rightarrow \frac{2}{\sqrt{7}} = a/b$

$\Rightarrow \sqrt{7} = 2b/a$

Since a & b are integers ⇒ $2b/a$ is a rational number

Hence $\sqrt{7}$ is also a rational number.

But, it is contradiction to our assumption.

Hence, our assumption is wrong.

Hence, $\sqrt{7}$ is a irrational number.

Hence proved.

ii) $3/(2\sqrt{5})$

→ Here, to prove $3/(2\sqrt{5})$ is a irrational number we use contradiction method.

Let us suppose, $3/(2\sqrt{5})$ is a rational number.

So we can write $3/(2\sqrt{5})$ as,

$\Rightarrow \frac{3}{2\sqrt{5}} = a/b$ where a & b are integers

$\Rightarrow \sqrt{5} = 3b/2a$

Since a & b are integers ⇒ $\frac{3b}{2a}$ is also a rational number.

⇒ $\sqrt{5}$ is a rational number.

But, it is contradiction to our assumption.

Hence, our assumption is wrong.

Hence, $3/2\sqrt{5}$ is a irrational number.

Hence proved.

iii) $(4+\sqrt{2})$

→ Here, to prove $(4+\sqrt{2})$ is a irrational number, we can use the method of contradiction.

Let us assume that, $(4+\sqrt{2})$ is a rational number.

So $(4+\sqrt{2})$ can be expressed as

$$(4+\sqrt{2}) = a/b$$

$$\Rightarrow \sqrt{2} = a/b - 4$$

$$\Rightarrow \sqrt{2} = (a-4b)/b$$

Since, a & b are integers $\Rightarrow \frac{(a-4b)}{b}$ is a rational number.

Hence, $\sqrt{2}$ is a rational number.

But, it is contradiction to our assumption.

Hence, our assumption is wrong.

Thus, $(4+\sqrt{2})$ is a irrational number.

Hence proved.

iv) $5\sqrt{2}$

→ Here, to prove $5\sqrt{2}$ is a irrational number we can use contradiction method.

Let us assume that, $5\sqrt{2}$ is a rational number.

So, $5\sqrt{2}$ can be written as

$5\sqrt{2} = a/b$ where a & b are integers.

$$\Rightarrow \sqrt{2} = a/5b$$

Since, a & b are integers $\Rightarrow a/5b$ is a rational number

$\Rightarrow \sqrt{2}$ is a rational number.

But it is contradiction to our assumption.

Hence, our assumption is wrong.

Thus, $5\sqrt{2}$ is a irrational number.

Hence proved.

Q.3.) Show that, $(2-\sqrt{3})$ is an irrational number.

→ Let us, to prove $(2-\sqrt{3})$ is an irrational number, we can use contradiction method.

Let us consider, $(2-\sqrt{3})$ is a rational number.

So, it can be written as

$(2-\sqrt{3}) = a/b$ where a & b are integers.

$\Rightarrow \sqrt{3} = 2 - a/b$

$\Rightarrow \sqrt{3} = (2b-a)/b$

Since, a & b are integers.

$\Rightarrow (2b-a)/b$ is a rational number.

$\Rightarrow \sqrt{3}$ is a rational number.

But, it is contradiction to our assumption.

Hence, our assumption is wrong.

So, $(2-\sqrt{3})$ is an irrational number.

Hence proved.

Q.4.) Show that $(3+\sqrt{2})$ is an irrational number.

→ Here, to show $(3+\sqrt{2})$ is an irrational number we can use a contradiction method.

Let us consider $(3+\sqrt{2})$ is a rational number.

So $(3+\sqrt{2})$ can be expressed as,

$\Rightarrow (3+\sqrt{2}) = a/b$ where a & b are integers

$\Rightarrow \sqrt{2} = a/b - 3$

$\Rightarrow \sqrt{2} = (a-3b)/b$

Since a & b are integers $\Rightarrow (a-3b)/b$ is a rational number

$\Rightarrow \sqrt{2}$ is a rational number.

But, it is contradiction to our assumption.

Hence, our assumption is wrong.

Thus, $(3+\sqrt{2})$ is an irrational number.

Hence proved.

Q.5) Show that $(4-5\sqrt{2})$ is an irrational number.

→ Here, to show $(4-5\sqrt{2})$ is a irrational number we can use contradiction method.

Let us assume, $(4-5\sqrt{2})$ is a rational number.

So, $(4-5\sqrt{2})$ is expressed as

$\Rightarrow (4-5\sqrt{2}) = a/b$

$\Rightarrow 5\sqrt{2} = 4 - a/b$

$\Rightarrow \sqrt{2} = (4b-a)/5b$

Since, a & b are integers. $\Rightarrow (4b-a)/5b$ is a rational number.

$\Rightarrow \sqrt{2}$ is a rational number

But it is contradiction to our assumption.

Hence, our assumption is wrong.

Thus, $(4-5\sqrt{2})$ is a irrational number.

Hence proved.

Q.6) Show that $(5-2\sqrt{3})$ is an irrational number.

→ Here, to show $(5-2\sqrt{3})$ is an irrational number, we can use the method of contradiction.

Let us suppose, $(5-2\sqrt{3})$ is a rational number.

So, $(5-2\sqrt{3})$ can be expressed as

$\Rightarrow (5-2\sqrt{3}) = a/b$ where a and b are integers

$2\sqrt{3} = a/b + 5$

$\sqrt{3} = (5b-a)/2b$

Since, a & b are integers $\Rightarrow (5b-a)/2b$ is a rational number.

$\Rightarrow \sqrt{3}$ is a rational number.

But, it is contradiction to our assumption.

Hence, our assumption is wrong.

Thus, $(5-2\sqrt{3})$ is an irrational number.

Hence proved.

Q.7) Prove that $(2\sqrt{3}-1)$ is an irrational number.

→ Here, to prove $(2\sqrt{3}-1)$ is an irrational number, we can use the method of contradiction.

Let us assume that, $(2\sqrt{3}-1)$ is a rational number.

So, it can be expressed as

$(2\sqrt{3}-1) = a/b$ where a & b are integers

$2\sqrt{3} = a/b + 1$

$\sqrt{3} = (a+b)/2b$

Since, a & b are integers ⇒ $(a+b)/2b$ is a rational number.

Hence, $\sqrt{3}$ is a rational number.

But it is contradiction to our assumption.

Hence, our assumption is wrong.

Thus, $(2\sqrt{3}-1)$ is an irrational number.

Q.8) Prove that $(2-3\sqrt{5})$ is an irrational number.

→ Here, to prove $(2-3\sqrt{5})$ is an irrational number, we can use the method of contradiction.

Let us assume that, $(2-3\sqrt{5})$ is a rational number.

So, it can be expressed as

$(2-3\sqrt{5}) = a/b$ where a, b are integers

⇒ $3\sqrt{5} = 2 - a/b$

⇒ $\sqrt{5} = (2b-a)/3b$

Since, a & b are integers ⇒ $(2b-a)/3b$ is a rational number.

⇒ $\sqrt{5}$ is a rational number.

But, this is contradiction to our assumption.

Hence, our assumption is wrong.

Thus, $(2-3\sqrt{5})$ is a irrational number.

Hence proved.

Q.9.) Prove that $(\sqrt{5}+\sqrt{3})$ is irrational number.

→ To prove $(\sqrt{5}+\sqrt{3})$ is a irrational number we can use a method of contradiction.

Let us consider $(\sqrt{5}+\sqrt{3})$ is a rational number.

So it can be expressed as,

$(\sqrt{5}+\sqrt{3}) = a/b$ where a and b are integers.

$\Rightarrow \sqrt{5} = a/b - \sqrt{3}$

squaring on both sides,

$5 = (a^2/b^2) + 3 - 2\sqrt{3}a/b$

$\Rightarrow (a^2/b^2) - 2 = 2\sqrt{3}a/b$

$(a/b) - 2b/a = 2/\sqrt{3}$

$(a^2-2b^2)/2ab = \sqrt{3}$

Since, a & b are integers $\Rightarrow (a^2-2b^2)/2ab$ is a rational number

Hence, (a $\sqrt{3}$ is a rational number.

But, it is contradiction to our assumption.

Hence, our assumption is wrong.

Thus, $(\sqrt{5}+\sqrt{3})$ is a irrational number.

Q.10.) Prove that $(\sqrt{2}+\sqrt{3})$ is irrational

→ To prove $(\sqrt{2}+\sqrt{3})$ is irrational, here we can used the method of contradiction.

Let us consider, $(\sqrt{2}+\sqrt{3})$ is a rational number.

So it can be expressed as

$(\sqrt{2}+\sqrt{3}) = a/b$ ∵ a & b are integers

$\sqrt{2} = a/b - \sqrt{3}$

$(\sqrt{2})^2 = [(a/b) - \sqrt{3}]^2$

$\Rightarrow 2 = (a^2/b^2) + 3 - (2\sqrt{3}\,a/b)$

$\Rightarrow (a^2/b^2) + 1 = (2\sqrt{3}a/b)$

$\Rightarrow (a/b) + (b/a) = 2\sqrt{3}$

$(a^2+b^2)/2ab = \sqrt{3}$

Since, a & b are integers $\Rightarrow (a^2+2b^2)/2ab$ is a rational number.

$\Rightarrow \sqrt{3}$ is a rational number.

Thus, it is contradiction to our assumption.

Hence, our assumption is wrong.

Hence, $(\sqrt{2}+\sqrt{3})$ is a irrational number.

Q.11.) Prove that for any prime integer p, $\sqrt{p}$ is an irrational number.

→ To prove $\sqrt{p}$ is an irrational number, we can use method of contradiction.

Let us assume that, $\sqrt{p}$ is a rational number.

So it can be expressed as,

$\sqrt{p} = a/b$ where, a & b are integers

$p^2 = a^2/b^2$

$pb = a^2/b$

p and b are integers $\Rightarrow pb = a^2/b$ will be an integer.

But, this is contradiction to our assumption.

Thus, $\sqrt{p}$ is an irrational number.

Hence proved.

Q.12.) If p, q are prime positive integers, prove that $(\sqrt{p}+\sqrt{q})$ is an irrational number.

→ To prove $(\sqrt{p}+\sqrt{q})$ is a irrational number, we use the method of contradiction.

Let us assume that $(\sqrt{p}+\sqrt{q})$ is a rational number.

Then, ∃ a co prime positive integers a and b such that,

$\sqrt{p}+\sqrt{q} = a/b$

$\sqrt{p} = (a/b) - \sqrt{q}$

$(\sqrt{p})^2 = [(a/b) - \sqrt{q}]^2$

$p = (a^2/b^2) + q - 2\sqrt{q}a/b$

$(a^2/b^2) - (p+q) = 2\sqrt{q}a/b$

$(a/b) - (p+q)b/a = 2\sqrt{q}$

$[a^2 - b^2(p+q)]/2ab = \sqrt{q}$

Since, a & b are integers ⇒ $[a^2 - b^2(p+q)]/2ab$ is rational.

Hence, $\sqrt{q}$ is rational number.

But, this is contradiction to our assumption.

So, our assumption is wrong.

Thus, $(\sqrt{p} + \sqrt{q})$ is an irrational number.

Exercise 1.6

Q.1) Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion.

1) 23/8

→ Given rational number is 23/8.

The denominator is 8. ⇒ $8 = 2^3 \times 5$

So, here denominator 8 of 23/8 can be expressed in the form $2^m \times 5^n$, where m & n are non-negative integers.

Hence, 23/8 has terminating decimal expansion.

Thus, the decimal expansion of 23/8 terminates after three places of decimal.

ii) 125/441

→ Here, the given rational number is 125/441.

The denominator is 441. ⇒ $441 = 3^2 \times 7^2$

Thus, the denominator 441 of 125/441 is not expressed in the form of $2^m \times 5^n$, where m, n are non-negative integers.

Thus, 125/441 has non-terminating repeating decimal expansion.

iii) 35/50

→ The given rational number is 35/50.

The denominator is 50. ⇒ $50 = 2 \times 5^2$

The denominator 50 of 35/50 is of the form $2^m \times 5^n$, where m, n are non-negative integers.

Hence, 35/50 has terminating decimal expansion.

And the decimal expansion of 35/50 terminates after two places of decimals.

iv) 77/210

→ The given rational number is 77/210.

The denominator is 210 ⇒ $210 = 2 \times 3 \times 5 \times 7$

Thus, here 210 which is denominator of 77/210 is not expressed in the form $2^m \times 5^n$, where m & n are non-negative integers.

Hence, 77/210 has non-terminating repeating decimal expansion.

v) $129/(2^2 \times 5^7 \times 7^{17})$

→ Here, the given rational number is $129/(2^2 \times 5^7 \times 7^{17})$.
The denominator is $(2^2 \times 5^7 \times 7^{17})$ which is not in the form $2^m \times 5^n$, where m & n are non-negative integers.
Thus, 125/441 has non-terminating repeating decimal expansion.

vi) 987/10500

→ Here, the given rational number is 987/10500.
which can be expressed as,

$$\frac{987}{10500} = \frac{47}{500}$$

Here, the denominator is 500. $\Rightarrow 500 = 2^2 \times 5^3$
Thus, the denominator 500 of 987/10500 is of the form $2^m \times 5^n$, where m, n are non-negative integers.
Hence, we can say that, 987/10500 has terminating decimal expansion.
And also, the decimal expansion of 987/10500 terminates after three places of decimals.

Q.2) Write down the decimal expansion of the following rational numbers by writing their denominators in the form of $2^m \times 5^n$, where m, & n are the non-negative integers.

i) 3/8

→ Given rational number is 3/8.
The denominator is 8 ⇒ $8 = 2^3$
Here, denominator 8 of 3/8 is expressed in the form $2^m \times 5^n$ where m = 3, n = 0.
Hence, the given number has terminating decimal expansion.

$$\therefore \frac{3 \times 5^3}{2^3 \times 5^3} = \frac{3 \times 125}{(2 \times 5)^3}$$

$$\frac{3\times5^3}{2^3\times5^3} = \frac{3\times125}{(2\times5)^3}$$

$$= \frac{375}{(10)^3}$$

$$= \frac{375}{1000}$$

$$\boxed{\frac{3\times5^3}{2^3\times5^3} = 0.375}$$

ii) 13/125

→ The given rational number is 13/125.

The denominator is $125 = 5^3$ which is expressed in the form $2^m\times5^n$, where $m=0$ and $n=3$.

Thus, the given number has terminating decimal expansion.

$$\therefore 13/125 = \frac{(13\times2^3)}{(125\times2^3)}$$

$$= \frac{104}{1000}$$

$$\boxed{13/125 = 0.104}$$

iii) 7/80

→ The given rational number is 7/80.

The denominator is $80 = 2^4\times5$ is of the form $2^m\times5^n$, where $m=4$ and $n=1$.

Thus, the given number has terminating decimal expansion.

$$\therefore \frac{7}{80} = \frac{7\times5^3}{2^4\times5\times5^3}$$

$$= \frac{7\times125}{(2\times5)^4} = \frac{875}{10^4} = \frac{875}{10000}$$

$$\boxed{\frac{7}{80} = 0.0875}$$

iv) $14588/625$

→ The given rational number is $14588/625$.

Here, the denominator is $625 = 5^4$ which is in the form $2^m \times 5^n$, where $m = 0$, $n = 4$.

Thus, the given number has terminating decimal expansion.

$$\therefore \frac{14588}{625} = \frac{14588 \times 2^4}{2^4 \times 5^4}$$

$$= \frac{14588}{5^4}$$

$$\boxed{\frac{14588}{625} = 23.3408}$$

v) $129/(2^2 \times 5^7)$

→ The given rational number is $129/(2^2 \times 5^7)$.

So, it can be written as $129/(2^2 \times 5^7)$.

Here, the denominator is expressed in the form $2^m \times 5^n$, where $m = 2$ and $n = 7$.

Thus, the given number has terminating decimal expansion

$$\therefore \frac{129}{2^2 \times 5^7} = \frac{129 \times 2^5}{2^2 \times 5^7 \times 2^5}$$

$$= \frac{129 \times 32}{(2 \times 5)^7}$$

$$= \frac{4182}{10^7}$$

$$= \frac{4128}{10000000}$$

$$\boxed{\frac{129}{2^2 \times 5^7} = 0.0004182}$$

vi) Q.3) Write the denominator of the rational number 257/5000 in the form $2^m \times 5^n$, where m, n are non-negative integers. Hence, write the decimal expansion, without actual division.

Soln:- Given rational number is 257/5000.

Here, the denominator of the given number is 5000.

$\Rightarrow 5000 = 2^3 \times 5^4$

Here, 5000 is expressed in the form $2^m \times 5^n$, where $m = 3, n = 4$.

$$\therefore \frac{257}{5000} = \frac{(257 \times 2)}{(5000 \times 2)}$$

$$= \frac{514}{10000}$$

$\boxed{\frac{257}{5000} = 0.0514}$ is the required decimal expansion.

Q.4. What can you say about the prime factorization of the denominators of the following rationals.

i) 43.123456789

→ Here, given decimal expansion 43.123456789 is terminating decimal expansion.

Hence, it's denominator will be in the form $(2^m \times 5^n)$, where m & n are non-negative integers.

ii) $43.\overline{123456789}$

→ Here, the given rational number is $43.\overline{123456789}$ which has non-terminating decimal expansion.

So, it's denominator has factors other than 2 and 5.

iii) $27.\overline{142857}$

→ Given rational number is $27.\overline{142857}$ which has non-terminating decimal expansion.

Hence, its denominator has factors other than 2 or 5.

iv) 0.120120012000120000...

→ Given number is 0.120120012000120000.... which has non-terminating decimal expansion.

So, it's denominator has factors other than 2 or 5.

Q.5) A rational number in it's decimal expansion is 327.7081. What can you say about the prime factors of q, when this number is expressed in the form p/q? Give reasons.

Soln:- The given rational number 327.7081 has a terminating decimal expansion its denominator should be of the form $2^m \times 5^n$ only, where m & n are non-negative integers.

Also, 327.7081 can be expressed as 3277081/10000 = p/q

$\Rightarrow q = 10000 = 2^3 \times 5^3$

Thus, here the prime factors of q has only factors of 2 and 5.