6 Square Roots & Cube Roots

Exercise : 6.1

Q.1 What will be the units digit of the square of the following numbers ?

i) 39 :

unit digit - 9

Square of 9 = 81

∴ Unit digit of square of 39 is 81

ii) 297 :

Unit digit - 7

Square of 7 = 49

∴ Unit digit of square of 297 is 9.

iii) 5125 :-

Unit digit - 5

Square of 5 = 25

∴ Unit digit of square of 5125 is 5.

iv) 7286 :-

Unit digit - 6

Square of 6 = 36

∴ Unit digit of square of 7286 is 6.

v) 8742 :

Unit digit – 2

Square of 2 = 4

∴ Unit digit of square of 8742 is 4.

Q2 Which of the following numbers are perfect squares ?

i) 121

121 = 11 × 11

∴ Hence, 121 is a perfect square.

ii) 136

136 = 136 is a not a perfect square

iii) 256

256 = 16 × 16

∴ 256 is a perfect square

iv) 321

∴ 321 is not a perfect square.

v) 600

∴ 600 is not a perfect square.

Q.3 The following numbers are not perfect squares. Give reasons ?

* The numbers that 2, 3, 7, 8 in unit place, are not perfect square.

i) 257

257 is not a perfect square because its unit digit is 7.

ii) 4592

4592 is not a perfect square because its unit digit is 2.

iii) 2433

2433 is not a perfect square because its unit digit is 3.

iv) 5050

5050 is not a perfect square because it can not expressed in the form of product of two equal integer.

v) 6098

6098 is not a perfect square because its unit digit is 8

Q4 Find wheather square of the following numbers are even or add

i) 431 :

unit digit - 1

Square of 1 = 1

∴ 1 is odd number.

∴ 431 is odd number

ii) 2826 :-

unit digit - 6

Square of 6 = 36

∴ 6 is a even number

∴ 2826 is a even number.

iii) 8204 :-

Unit digit - 4

Square of 4 = 16

∴ 6 is a even number

∴ 8204 is even number.

iv) 17779

Unit digit - 9

Square of 9 = 81

1 is odd number.

∴ 17779 is a odd number.

v) 99998

Unit digit - 8

Square of 8 = 64

4 is even number.

99998 is a even number.

Q.5 How many numbers lie between square of the following numbers:

i) 25, 26

$25^2 = 625$

$26^2 = 676$

$\therefore 676 - 625 = 51$

$n - 1 = 51 - 1 = 50$

$\therefore$ 50 numbers are in between square of 25 & 26

ii) 56, 57

$56 = 3136$

$57 = 3249$

$3136 - 3249 = 113$

$n - 1 = 113 - 1 = 112$

$\therefore$ 112 numbers are in between squares of 56 & 57.

iii) 107 ; 108

$107^2 = 11449$

$108^2 = 11664$

$11664 - 11449 = 215$

$n - 1 = 215 - 1 = 214$

$\therefore$ 214 are numbers are in between squares of 107, 108.

Q. 6 Without adding, find the sum of the following numbers.

i) $1 + 3 + 5 + 7 + 9 =$

1, 3, 5, 7, 9 are odd numbers.

Sum of first n odd numbers $= n^2$

$n = 5$

$n^2 = 5^2 = \underline{\underline{25}}$

$\therefore$ Sum of $1 + 3 + 5 + 7 + 9$ is 25.

ii) $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 =$

$n = 9$

$n^2 = 9^2 = \underline{\underline{81}}$

$\therefore$ Sum of $1+3+5+7+9+11+13+15+17$ is 81.

iii) $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 + 25$

$n = 13$

$n^2 = 13^2 = \underline{\underline{169}}.$

$\therefore$ Sum of $1+3+5+7+9+11+13+15+17+19+21+23+25$ is 169.

Excercise : 6.2

Q.1. Find the square root of the following numbers by prime Factorization method.

i) 441

3	441
3	147
7	49
7	7
	1

$441 = \underline{3 \times 3} \times \underline{7 \times 7}$

$= 3 \times 7$

$= \underline{21}$

∴ Square root of 441 is 21.

ii) 784

2	784
2	392
2	196
2	98
7	49
7	7
	1

$784 = \underline{2 \times 2 \times 2 \times 2} \times \underline{7 \times 7}$

$= 2 \times 2 \times 7$

$= \underline{28}$

∴ Square root of 784 is 28.

iii) 4096

2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$$4096 = \frac{2\times2\times2\times2\times2\times2}{2\times2\times2\times2\times2\times2}$$

$$= 2\times2\times2\times2\times2\times2$$

$$= 64$$

$\therefore$ Square of 4096 is 64.

iv) 7056

2	7056
2	3528
2	1764
2	882
3	441
3	147
7	49
7	7
	1

$7056 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7}$

$= 2 \times 2 \times 3 \times 7$

$= 84$

$\therefore$ Square of 7056 is 84.

Q.2 Find the smallest number by which 3645 must be multiplied to get a perfect square.

3	3645
3	1215
3	405
3	135
3	45
3	15
5	5
	1

$3645 = \underline{3 \times 3} \times \underline{3 \times 3} \times \underline{3 \times 3} \times 5$

$= 3 \times 3 \times 3 \times 5$

∴ 3, 3, 3 are in pair, 5 is not in a pair

So 3645 must be multiplied by 5 to get perfect square

Q.3 Find the smallest number by which 2400 is to be multiplied to get a perfect square & also find the square root of resulting number.

Ans

2	2400
2	1200
2	600
2	300
2	150
3	75
5	25
5	5
	1

$2400 = \underline{2 \times 2} \times \underline{2 \times 2} \times 2 \times 3 \times \underline{5 \times 5}$

$= 2 \times 2 \times 5 \times 2 \times 3$

∴ 2, 2, 5 are in pairs & 2, 3 are not in pairs.

∴ For perfect square, we need to multiply 2400 into 3 & 2.

$2400 \times 3 \times 2 = 2400 \times 6 = 14400$

So we have to find square root of 14400.

2	14400
2	7200
2	3600
2	1800
2	900
2	450
3	225
3	75
5	25
5	5
	1

$$14400 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$$
$$= 2 \times 2 \times 2 \times 3 \times 5$$
$$= 120$$

∴ Square root of a number 14400 is 120.

Q.4 Find the smallest number by which 7776 is to be divided to get a perfect square.

2	7776
2	3888
2	1944
2	972
2	486
3	243
3	81
3	27
3	9
3	3
	1

$$7776 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3$$
$$= 2 \times 2 \times 2 \times 3 \times 3 \times 3$$

Here, 2, 2, 3, 3 are in pairs & 2, 3 are not in pairs.

So we have to divide 7776 by 3 × 2.

$$= \frac{7776}{3 \times 3} = \frac{3888}{3} = \frac{1296}{}$$

$3 \times 2 = 6$

∴ 6 is the number we should get a perfect square

Q5 1521 trees are planted in a garden in such a way that there are so many trees in each rows as there are rows in the garden. Find the no. of rows & no. of trees in each row.

Ans:- Suppose the no. of rows is x.
∴ So the no. of trees in each row will also equal to x.

∴ Total trees = $x \times x = x^2$
Given ∴ total trees = 1521

$x^2 = 1521$

So we have to find square root of 1521.

3	1521
3	507
13	169
13	13
	1

$\therefore 1521 = 3 \times 3 \times 13 \times 13$

$= 3 \times 13$

$= 39$

$x = 39$

$\therefore$ So the no. of trees is 39 & no. of rows is 39.

Q.6 A school collected ₹ 2601 as fees from its students. If fee paid by each student & number students in the school were equal, how many students were there in the school?

Ans:- Suppose the no. of students in a school is x.

So fees paid by each students is x.

$\therefore x \times x = x^2$

$\therefore x^2 = 2601$

∴ So we have to find square root of 2601.

3	2601
3	867
17	289
17	17
	1

$2601 = \underline{3 \times 3} \times \underline{17 \times 17}$

$= 3 \times 17$

$= \underline{\underline{51}}$

$x = 51$

∴ 51 students were there in school.

Q.7 The product of two numbers is 1296. If one number is 16 times the other, find the two numbers.

Suppose the number is x & other number is 16x.

$x \times 16x = 1296$

$16x^2 = 1296$

$$x^2 = \frac{1296}{16}$$

$$x^2 = 81$$

$$x = \sqrt{81}$$

$$x = \underline{9}$$

$$16x = 16 \times 9 = 144$$

$\therefore$ The two numbers are 9 & 144.

Q. 8 7921 soliders sat in an auditorium in such a way that there are as many soldiers in a row as there are rows in the auditorium. How many rows are there in auditorium?

Ans:- Suppose no. of soldiers in each is x.

So no. of rows in the auditorium is x.

Total soldiers = 7921

$$x \times x = 7921$$

$$x^2 = 7921$$

So we have to find square root of 7921.

89	7921
89	89
	1

$7921 = 89 \times 89$

$= 89$

$x = 89$

$\therefore$ No. of rows in auditorium is 89.

Q.9 The area of a square field is $5184\ m^2$. Find the area of rectangular field whose perimeter is equal to the perimeter of square field & whose length is twice of its breadth.

Given :-

Area of square $= 5184\ m^2$

Perimeter of rectangular field = Perimeter of square field & length = twice breadth.

Area of square $= (\text{side})^2$

$5184 = (\text{side})^2$

Side $= \sqrt{5184}$

2	5184
2	2592
2	1296
2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

$5184 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3}$

$= 2 \times 2 \times 2 \times 3 \times 3$

$= 72$

side $= 72$

Perimeter of square $= 4 \times \text{side}$

$= 4 \times 72$

$= \underline{288}$ m

Exercise :- 6 : 3

Q.1 Find the square root of the following by division method.

i) 1089

	3 3
3	$\overline{10}\,\overline{89}$
	9
63	189
	189
	0

$\sqrt{1089} = 33$

ii) 2304

	4 8
4	$\overline{23}\,\overline{04}$
	-16
88	704
	-704
	0

$\sqrt{2304} = 48$

iii) 7744

	8 8
8	$\overline{77}\,\overline{44}$
	-64
168	1344
	-1340
	0

$\sqrt{7744} = 88$

iv) 6084

	7 8
8	$\overline{60}\,\overline{84}$
	-49
148	1184
	-1184
	0

$\sqrt{6084} = 78$

iii) 68.89

	8.3
8	$\overline{52}.\overline{89}$
	-64
163	4.89
	-4.89
	0

iv) 24.64

	9.2
9	$\overline{24}.\overline{64}$
	-81
182	3.64
	-3.64
	0

$\sqrt{68.89} = 8.3$ $\sqrt{24.64} = 9.2$

Q.3 Find the least number that is to be subtracted from 4000 to make it perfect square

Ans:- By division method

	63
6	4000
	-36
123	400
	-369
	31

∴ We need to subtract 31 from 4000

4000 - 31 = 3969

So the side of the square root

is 33

$$\begin{array}{r} 33 \\ \hline 33\overline{)1089} \\ -99 \\ \hline 99 \quad 99 \\ -99 \\ \hline 0 \end{array}$$

$\therefore \sqrt{1089} = 33$

The ans number is 1089

Q. Find the length of the side of a square whose area is 4489 sq.cm.

Sol:- Given -

Area = 4489 sq.cm

Area of square = $(\text{side})^2$

$4489 = (\text{side})^2$

$\text{side} = \sqrt{4489}$

$= 67$

Q.5 A gardner wishes to plant 8289 plants in the form of a square & found that there were 8 plants left How many plants were planted in each row.

Ans:- 8 plants were left from 8289 plants

$8289 - 8 = 8281$

Suppose there are x plants in a row & there are x no. of rows.

$\therefore \quad x \times x = 8281$

$x^2 = 8281$

$x = \sqrt{8281}$

$x = 91$

$\therefore$ There are 91 plants in each row.

Q.6 Find the least perfect square with four digits.

The least number with four digits is 1000.

We find square root of 1000 by ~~fact~~ division method.

	31.6
3	$\overline{10}\,\overline{00}.\overline{00}$
	-9
61	100
	-61
626	3900
	-3756
	144

$\sqrt{1000} = 31.6$ which is not a perfect square.

$\therefore$ So the no. after 31.6 is 32.

$32^2 = 32 \times 32 = 1024$

$\therefore$ 1024 is the smallest four digit number which is a perfect square.

Q.7 Find the least number which must be added to 6412 to make it a perfect square ?

Ans:- By division method,

	80
8	6412
	-64
160	0012
	- 0
	12

6412 is not a perfect square.

The no. after 80 is 81.

$81^2 = 81 \times 81 = 6561$

$\therefore$ $6561 - 6412 = 149$

$\therefore$ 149 must be added to 6412 to make a perfect square.

Q.8 Estimate the value of following numbers to nearest whole number

i) $\sqrt{97}$

97 lies between 9^2 & 10^2.

$9^2 = 81$; $10^2 = 100$

$9 < \sqrt{91} < 100$

$\therefore \sqrt{97} \approx 9$

ii) $\sqrt{250}$

250 lies between 15^2 & 16^2.

$15^2 = 225$; $16^2 = 256$

$15 < \sqrt{250} < 16$

$\therefore \sqrt{250} \approx 15.$

iii) 780

780 lies between 27^2 & 28^2

$27^2 = 729$; $28^2 = 784$

$\sqrt{729} < \sqrt{780} < \sqrt{784}$

$27 < \sqrt{780} < 28$

$\therefore \sqrt{780} \approx 27.$

Exercise : 6.4

Q.1 Find the cube of following numbers.

i) 8

$8^3 = 8 \times 8 \times 8 = 512$

ii) 16

$16^3 = 16 \times 16 \times 16 = 256 \times 16 = 4096$

iii) 21

$21^3 = 21 \times 21 \times 21 = 441 \times 21 = 9261$

iv) 30

$30^3 = 30 \times 30 \times 30 = 900 \times 30 = 27000$

Q.2 Test wheather the given numbers are perfect cubes or not.

i) 243

3	243
3	81
3	27
3	9
3	3
	1

$243 = \underline{3 \times 3 \times 3} \times 3 \times 3$

$= 3 \times 3 \times 3$

$\therefore$ 3 is not in triplet.

$\therefore$ 243 is not a pefect cube.

ii) 516

2	516
2	258
3	129
43	43
	1

$516 = 2 \times 2 \times 3 \times 43$

$\therefore$ 2, 3, 43 are not in triplet.

516 is not a perfect cube.

iii) 729

3	729
3	243
3	81
3	27
3	9
3	3
	1

$729 = \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}$

$= 3 \times 3$

$= 9$

$\therefore$ 3 & 3 is in triplet.

$\therefore$ 729 is a perfect cube.

iv) 8000

2	8000
2	4000
2	2000
2	1000
2	500
2	250
5	125
5	25
5	5
	1

$8000 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5$

$= 2 \times 2 \times 5 = 4 \times 5$

$= 20$

$\therefore$ 2, 2 & 5 are in triplet.

$\therefore$ 8000 is a perfect cube.

v) 2700

2	2700
2	1350
3	675
3	225
3	75
5	25
5	5
	1

$2700 = 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5$

$= 2 \times 2 \times 3 \times 5$

$\therefore$ 2, 3 & 5 are not in triplet.

$\therefore$ 2700 is not a perfect cube.

3	7803
3	2601
3	867
17	289
17	17
	1

$7803 = \underline{3 \times 3 \times 3} \times 17 \times 17$

$= 3 \times 17 \times 17$

$\therefore$ 17 is left in triplet

$\therefore$ 17 should be multiplied to 7803 to get perfect cube number.

Q.5 Find the smallest number by which 8640 must be divided so that the quotient is a perfect cube ?

2	8640
2	4320
2	2160
2	1080
2	540
3	270
3	90
3	30
2	10
5	5
	1

$8640 = \underline{2 \times 2 \times 2 \times 2 \times 2 \times 2}$

$= \underline{3 \times 3 \times 3} \times \underline{5}$

$\therefore$ 2, 3, 2 are in triplet & 5 is left in triplet.

$\therefore$ By 5 to 8640 must be divided.

5. How many a cube of edge 2 cm can be made of dimensions 2 cm × 8 cm × 32 cm. How many minimum number of such cubes will be needed to form cube?

Ans. 16 cm

Side of cube = 2 cm

Cube's dimensions = 2 cm × 8 cm × 32 cm

Volume of cube = $(\text{side})^3$

$= (2)^3$

$= 2 \times 2 \times 2$

$= 4 \times 2$

$= \underline{8}\ cm^3$

Volume of cuboid = $2 \times 8 \times 32$

$= 16 \times 32$

$= \underline{512}\ cm^3$

No. of cubes needed to form cuboid =

$$\frac{\text{Volume of cuboid}}{\text{Volume of cube}} = \frac{512\ cm^3}{8\ cm^3}$$

$$= \frac{512}{8} = \frac{64}{1} = \underline{64}$$

∴ 64 cubes are needed to form cuboid.

Q 7 Find the smallest prime number dividing the sum $3^{11} + 5^{13}$.

The sum of two odd numbers is even.

$\therefore$ $3^{11} + 5^{13}$ = Even

$\therefore$ Any Even no. is divisible by 2

$\therefore$ 2 divides $3^{11} + 5^{13}$.

Exercise 6.5

Q.1 Find the cube root of the following numbers by prime factorization method

i) 343

7	343
7	49
7	7
	1

$343 = 7 \times 7 \times 7$

$= 7$

$\sqrt[3]{343} = 7$

ii) 729

3	729
3	243
3	81
3	27
3	9
3	3
	1

$729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$= 3 \times 3$

$= 9$

$\sqrt[3]{729} = 9$

iii) 1331

11	1331
11	121
11	11
	1

$1331 = 11 \times 11 \times 11$

$= 11$

$\therefore \sqrt[3]{1331} = \underline{\underline{11}}$

iv) 2744

2	2744
2	1372
2	686
7	343
7	49
7	7
	1

$2744 = \underline{2 \times 2 \times 2} \times \underline{7 \times 7 \times 7}$

$= 2 \times 7$

$= 14$

$\therefore \sqrt[3]{2744} = \underline{\underline{14}}$

Q. 2 Find the cube root of the following numbers through estimation.

i) 512

Unit place digit = 2

$2^3 = 2 \times 2 \times 2 = 8$

There is no second group.

$\sqrt{512} = 8$

ii) 2197

$\overline{2}\ \overline{197}$

Unit place = 7

$7^3 = 7 \times 7 \times 7 = 49 \times 7 = 343$

$\therefore$ Unit digit = $\underline{\underline{3}}$

$\therefore$ second group = 2

$1^3 = 1$ & $2^3 = 8$

$1^3 < 2 < 2^3$

$\therefore$ $\underline{\underline{1}}$ is taken at tens place

$\therefore \sqrt[3]{2197} = \underline{\underline{13}}$

iii) 3375

$\overline{3}\ \overline{375}$

Unit place = 5

$5^3 = 5 \times 5 \times 5 = 25 \times 5 = 125$

Unit digit = 5

second group = 3

$1^3 < 3 < 2^3$

$\therefore$ 1 becomes tens place

$\therefore \sqrt[3]{3375} = \underline{\underline{15}}$

iv) 5832

5832

unit place = 2

$2^3 = 2 \times 2 \times 2 = 4 \times 2 = \underline{\underline{8}}$

Unit digit = $\underline{\underline{8}}$

second group = 5

$1^3 < 5 < 2^3$

$1^3 < 5 < 8$

$\therefore$ Tens place = $\underline{\underline{1}}$

$\therefore \sqrt[3]{5832} = \underline{\underline{18}}$

Q.3 State true or False

i) Cube of an even number is an odd number

False (Cube of even number is even number)

ii) A pefect cube may end with two zeros

False

iii) If a number ends with 5 then its cubes ends with 5.
True

iv) Cube of a number ending with zero has three zeros at its right
True

v) The cube of single digit number may be a single digit number
~~False~~ True

vi) There is no perfect cube which ends with 8.
False (There are many cubes which ends with 8)

vii) The cube of a two digit number may be a three digit number.
False (The smallest two digit number is 10 & $10^3 = 1000$ which has 4 digits in it)

Q.4 Find the two digit number which is a square number & also a cubic number.

64 is the two digit number

$4^3 = 4 \times 4 \times 4 = 16 \times 4 = \underline{64}$

$8^2 = 8 \times 8 = 64$

$\therefore$ 64 is the two digit no. which is a square number & also a cubic number.