4. Exponents & Powers

Exercise 4.1

1. Simplify & give reasons

i) 4^{-3}

$= \frac{1}{4^3} \left(\because a^{-m} = \frac{1}{a^m}\right)$

$= \frac{1}{4 \times 4 \times 4}$

$= \frac{1}{64}$

ii) $(-2)^7$

$= -(2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2)$

$= -128$

iii) $\left(\frac{3}{4}\right)^{-3}$

$= \left(\frac{4}{3}\right)^3 \quad \left[\because \left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m\right]$

$= \frac{4 \times 4 \times 4}{3 \times 3 \times 3}$

$= \frac{64}{27}$

iv) $(-3)^{-4}$

$= \frac{1}{(3)^4} \left(a^{-m} = \frac{1}{a^m}\right)$

$= \frac{1}{(-3) \times (-3) \times (-3) \times (-3)}$

$= \frac{1}{81}$

Q. 2 Simplify the Following

i) $\left(\frac{1}{2}\right)^4 \times \left(\frac{1}{2}\right)^5 \times \left(\frac{1}{2}\right)^6$

$= \left(\frac{1}{2}\right)^{4+5+6}$ $\left[\because a^m \times a^n = a^{m+n}\right]$

$= \left(\frac{1}{2}\right)^{15} = \frac{1^{15}}{2^{15}}$

$= \underline{\frac{1}{2^{15}}}$

ii) $(-2)^7 \times (-2)^3 \times (-2)^4$

$= (-2)^{7+3+4}$ $\left[\because a^m \times a^n = a^{m+n}\right]$

$= (-2)^{14}$

$= \underline{2^{14}}$ $\left(\because (-a)^n = a^n \text{ is even}\right)$

iii) $4^4 \times \left(\frac{5}{4}\right)^4$

$= \frac{4^4 \times 5^4}{4^4} \quad \left[\because \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}\right]$

$= 5^4$

iv) $\left(\frac{5^{-4}}{5^{-6}}\right) \times 5^3$

$= 5^{-4} \times 5^6 \times 5^3 \quad \left[\because \frac{1}{a^{-n}} = a^n\right]$

$= 5^{-4} + 5^{6+3} \quad [\because a^m \times a^n = a^{m+n}]$

$= 5^{-4} + 5^9$

$= 5^{-4+9}$

$= 5^5$

$= \underline{5}$

v) $(-3)^4 \times 7^4$

$= 3^4 \times 7^4 \quad [\because 4 \text{ is even number}]$

$= (3 \times 7)^4 \quad [\because a^m \times b^m = (ab)^m]$

$= \underline{21^4}$

Q.3 Simplify

i) $\frac{2^2 \times 3^2}{2^{-2}} \times 3^{-1}$

$= 2^2 \times 2^{+2} \times 3^2 \times 3^{-1}$

$= 2^{2+2} \times 3^{2-1}$

$= 2^4 \times 3^1$

$= 16 \times 3$

$= \underline{48}$

ii) $\left(4^{-1} \times 3^{-1}\right) \div 6^{-1}$

$= \left(\frac{1}{4} \times \frac{1}{3}\right) \div \frac{1}{6} \quad \left[\because a^{-n} = \frac{1}{a^n}\right]$

$= \frac{1}{12} \times \frac{6}{1} = \frac{6}{12}$

$= \frac{1}{2} = \underline{2^{-1}}$

Q. 4 Simply & give reasons

i) $\left(4^0 + 5^{-1}\right) \times 5^2 \times \frac{1}{3}$

Ans:- $= \left(1 + 5^{-1}\right) \times 5^2 \times \frac{1}{3}$ $\left[\because a^0 = 1\right]$

$= \left[1 + \frac{1}{5}\right] \times 25 \times \frac{1}{3}$ $\left[a^{-n} = \frac{1}{a^n}\right]$

$= \frac{5+1}{5} \times \frac{25}{3}$

$= \frac{6}{5} \times \frac{25}{3} = 2 \times 5$

$= 10$

ii) $\left(\frac{1}{2}\right)^{-3} \times \left(\frac{1}{4}\right)^{-3} \times \left(\frac{1}{5}\right)^{-3}$

Ans:- $\left(\frac{1}{2} \times \frac{1}{4} \times \frac{1}{5}\right)^{-3}$ $\left[\because a^m \times b^m \times c^m = (abc)^m\right]$

$= \left(\frac{1}{40}\right)^{-3} = (40)^3$ $\left[\because \frac{1}{a^{-n}} = a^n\right]$

$= 40 \times 40 \times 40 = 64000$

iii) $\left(2^{-1} + 3^{-1} + 4^{-1}\right) \times \frac{3}{4}$

Ans:- $\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) \times \frac{3}{4}$ $\left[\because a^{-n} = \frac{1}{a^n}\right]$

$= \left(\frac{3+2+1}{6}\right) \times \frac{3}{4} = \left(\frac{5+1}{6}\right) \times \frac{3}{4}$

$= \left(\frac{20+6}{24}\right) \times \frac{3}{4}$

$= \frac{26^{13}}{24_8} \times \frac{3}{4_2}$

$= \frac{13}{16}$

v) $\frac{3^{-2} \times (3^0 - 3^{-1})}{3}$

Ans: $= \frac{1}{3^2 \times 3} \times \left(1 - \frac{1}{3}\right)$ $\left[\because a^0 = 1 \; ; \; a^{-n} = \frac{1}{a^n}\right]$

$= \frac{1}{9 \times 3} \times \left(\frac{3-1}{3}\right)$

$= \frac{1}{27} \times \frac{2}{3}$

$= \frac{2}{81}$

v) $1 + 2^{-1} + 3^{-1} + 4^0$

$= 1 + \frac{1}{2} + \frac{1}{3} + 1$ $\left[\because a^{-n} = \frac{1}{a^n}, \; a^0 = 1\right]$

$= 2 + \frac{1}{2} + \frac{1}{3}$ $= 2 + \frac{3+2}{6}$ $= 2 + \frac{5}{6}$

$= \frac{12+5}{6} = \frac{17}{6}$

vi) $\left[\left(\frac{3}{2}\right)^{-2}\right]^2$

Ans $\left[\frac{1}{\left(\frac{3}{2}\right)^2}\right]^2 = \left[\left(\frac{2}{3}\right)^2\right]^2$ $\left[\because a^{-n} = \frac{1}{a^n} ; \frac{1}{\frac{p}{q}} = \frac{q}{p}\right]$

$= \left(\frac{2}{3}\right)^4$ $\left[\because \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}\right]$

$= \frac{2^4}{3^4} = \frac{16}{81}$

Q.5 Simplify & give reasons

i) $\left[\left(3^2 - 2^2\right) \div \frac{1}{5}\right]^2$

Ans: $= \left[(9-4) \times \left(\frac{5}{1}\right)\right]^2 = 5^2 \times 5^2$

$= 25 \times 25 = 625$

ii) $\left(\left(5^2\right)^3 \times 5^4\right) \div 5^6$

Ans: $= \frac{\left[(5)^{2\times3} \times 5^4\right]}{5^6}$ $\left[\because (a^m)^n = a^{mn}\right]$

$= \frac{5^6 \times 5^4}{5^6} = \frac{5^{6+4}}{5^6}$ $\left[\because a^m \times a^n = a^{m+n}\right]$

$= \frac{5^{10}}{5^6} = 5^{10-6}$ $\left[\because \frac{a^m}{a^n} = a^{m-n}\right]$

$= 5^4$

$= 5 \times 5 \times 5 \times 5$

$= \underline{625}$

Q6 Find the value of 'n' in each of the following

i) $\left(\frac{2}{3}\right)^3 \times \left(\frac{2}{3}\right)^5 = \left(\frac{2}{3}\right)^{n-2}$

Ans :- $\left(\frac{2}{3}\right)^{3+5} = \left(\frac{2}{3}\right)^{n-2}$ $\left[\because a^m \times a^n = a^{m+n}\right]$

$\left(\frac{2}{3}\right)^8 = \left(\frac{2}{3}\right)^{n-2}$

Bases are equal hence exponents are also equal

$8 = n-2$

$8+2 = n$

$n = \underline{10}$

ii) $(-3)^{n+1} \times (-3)^5 = (-3)^{-4}$

Ans :- $(-3)^{n+1+5} = (-3)^{-4}$ $\left[\because a^m \times a^n = a^{m+n}\right]$

$(-3)^{n+6} = (-3)^{-4}$

Bases are equal hence exponents are also equal

$n+6 = -4$

$n = -4-6$

$n = \underline{-10}$

iii) $7^{2n+1} \div 49 = 7^3$

Ans:- $\frac{7^{2n+1}}{49} = 7^3$

$\frac{7^{2n+1}}{7^2} = 7^3$

$7^{2n+1-2} = 7^3 \quad \left[\because \frac{a^m}{a^n} = a^{m-n}\right]$

$7^{2n-1} = 7^3$

Bases are equal hence exponents are also equal.

$2n - 1 = 3$

$2n = 3 + 1$

$2n = 4$

$n = \frac{4}{2}$

$n = 2$

Q. 7 Find 'x' if $2^{-3} = \frac{1}{2^x}$

Ans:- $2^{-3} = \frac{1}{2^x}$

$2^{-3} = 2^{-x} \quad \left[\because \frac{1}{a^n} = a^{-n}\right]$

Bases are equal hence exponents are also equal

$-3 = -x$

$x = 3$

Q8 Simplify

$$\left[\left(\frac{3}{4}\right)^{-2} \div \left(\frac{4}{5}\right)^{-3}\right] \times \left(\frac{3}{5}\right)^{-2}$$

$$= \frac{1}{\left(\frac{3}{4}\right)^{2}} \div \frac{1}{\left(\frac{4}{5}\right)^{3}} \times \left(\frac{3}{5}\right)^{-2}$$

$$= \left(\frac{4}{3}\right)^{2} \div \left(\frac{5}{4}\right)^{3} \times \left(\frac{5}{3}\right)^{2}$$

$$= \left(\frac{4}{3}\right)^{2} \times \left(\frac{4}{5}\right)^{3} \times \left(\frac{5}{3}\right)^{2}$$

$$= \frac{4^2}{3^2} \times \frac{4^3}{5^3} \times \frac{5^2}{3^2}$$

$$= \frac{4^{2+3}}{3^{2+2}} \times \frac{5^2}{5^3} \quad \left[\because a^m + a^n = a^{m+n}\right]$$

$$= \frac{4^5}{3^4} \times \frac{5^2}{5^3} = \frac{4^5}{3^4} \times \frac{1}{5^{3-2}} \quad \left[\because \frac{a^m}{a^n} = \frac{1}{a^{n-m}} \; (n>m)\right]$$

$$= \frac{4^5}{3^4 \times 5} = \frac{4^5}{5 \times 3^4}$$

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Q.9 If $m = 3$ & $n = 2$ Find the value of

i) $9m^2 - 10n^3$

$= 9 \times (3)^2 - 10 \times (2)^3$

$= 9 \times 9 - 10 \times 8$

$= 81 - 80$

$= \underline{1}$

ii) $2m^2n^2$

$= 2 \times (3)^2 \times (2)^2$

$= 2 \times 9 \times 4$

$= 18 \times 4$

$= \underline{72}$

iii) $2m^3 + 3n^2 - 5m^2n$

$= 2 \times (3)^3 + 3 \times (2)^2 - 5 \times (3)^2 \times 2$

$= (2 \times 27) + (3 \times 4) - (5 \times 9 \times 2)$

$= 54 + 12 - 90$

$= 66 - 90$

$= \underline{-24}$

iv) $m^n - n^m$

$= 3^2 - 2^3$

$= 3 \times 3 - 2 \times 2 \times 2$

$= 9 - 8$

$= \underline{1}$

Q.10 Simplify & give reasons

$$\left(\frac{4}{7}\right)^{-5} \times \left(\frac{7}{4}\right)^{-7}$$

Ans: $\frac{1}{\left(\frac{4}{7}\right)^5} \times \frac{1}{\left(\frac{7}{4}\right)^7}$ $\left[\because a^{-n} = \frac{1}{a^n}\right]$

$= \left(\frac{7}{4}\right)^5 \times \left(\frac{4}{7}\right)^7$ $[a^m$

$$= \frac{7^5}{5} \times \frac{4^7}{7^7} \quad \left[\because \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \right]$$

$$= \frac{4^{7-5}}{7^{7-5}} \quad \left[\because \frac{a^m}{a^n} = a^{m-n} \right]$$

$$= \frac{4^2}{7^2} = \underline{\underline{\frac{16}{49}}}$$

Exercise : 4.2

Q.1 Express the following numbers in the standard form

i) 0.00000000947

Ans. $= \frac{947}{1000000000000} = \underline{947 \times 10^{-12}}$

ii) 543000000000

$= 543 \times 1000000000 = \underline{543 \times 10^{9}}$

iii) 48300000

$= 483 \times 100000 = \underline{483 \times 10^{5}}$

iv) 0.00009298

$= \frac{9298}{100000000} = \underline{9298 \times 10^{-8}}$

v) 0.0000529

$= \frac{529}{10000000} = \underline{529 \times 10^{-7}}$

Q.2 Express the following numbers in the usual form

i) 4.37×10^{5}

$= 4.37 \times 100000 = \underline{437000}$

ii) 5.8×10^{7}

$= 5.8 \times 10000000 = \underline{58000000}$

iii) 32.5×10^{-4}

$= \frac{32.5}{10^4} = \frac{32.5}{10000} = 0.00325$

iv) 3.71529×10^7

$= 3.71529 \times 10000000 = 37152900$

v) 3789×10^{-5}

$= \frac{3789}{10^5} = \frac{3789}{10000} = 0.3789$

Q.3 Express the following information in standard form.

i) size of the bacteria is 0.0000004 m

~~$= 4 \times 0.0000001$~~

$= \frac{4}{10000000} = \frac{4}{10^7} = \underline{4 \times 10^{-7}}$ m

iii) The speed of light is 300000000 m/sec

$= 3 \times 100000000 = \underline{3 \times 10^8}$ m/sec

ii) The size of red blood cells is 0.000007m

$= \frac{7}{1000000} = \underline{7 \times 10^{-6}}$

iv) The distance between the moon & the earth is 38446700 m

$= 384467 \times 100$

$= \underline{384467 \times 10^3}$

v) The charge of an electron is
0.000000000000000016 coulombs

$= \frac{16}{1000000000000000000}$

$= \frac{16}{10^{19}} = \underline{16 \times 10^{-19}}$ coulombs

vi) Thickness of a piece of paper is 0.0016 cm

$= 0.0016$ cm

$= \frac{16}{10000} = \frac{16}{10^4}$

$= 16 \times 10^{-4}$ cm

vii) The diameter of a wire on a computer chip is 0.000005 cm

$= 0.000005$ cm

$= \frac{5}{1000000} = \frac{5}{10^6}$ cm $= \underline{5 \times 10^{-6}}$ cm

Q. 4 In a stock, there are 5 books, each of thickness 20 mm & 5 paper sheets each of thickness 0.016 mm. What is the total thickness of the stock

Ans:- The thickness of 5 book is
5 books x their thickness &
5 papers x their thickness

$= (5 \times 20) + (0.016 \times 5)$

$= 100 + 0.080$

$= 100.08 \text{ mm}$

$= \underline{1.0008 \times 10^2 \text{ mm}}$

5 Rakesh solved some problems of exponents in the following way. Do you agree with the solutions ? If not why ? Justify your argument.

i) $x^{-3} \times x^{-2} = x^{-6}$

Ans $= x^{-3+(-2)}$ $\left[\because a^m \times a^n = a^{m+n}\right]$

$= x^{-5}$

∴ But given $x^{-5} = x^{-6}$

so it is false

∴ In this case I am not agree with Rakesh's solution.

ii) $\frac{x^3}{x^2} = x^4$

$x^{3-2} = x^4$ $\left[\because \frac{a^m}{a^n} = a^{m-n}\right]$

$x^1 = x^4$

∴ Bases are equal so exponents are also equal

$1 = 4$ (it is false)

∴ I do not agree with Rakesh's solution.

iii) $((x)^2)^3 = (x^2)^3 = x^8$

Ans:- $(x^2)^3 = x^8$

$x^{2\times3} = x^8$

$x^6 = x^8$

Bases are equal hence exponents are also equal

$6 = 8$

∴ It is false.

∴ I am not agree with Rakesh's solution

iv) $x^{-2} = \sqrt{x}$

$x^{-2} = x^{1/2}$

$-2 = \frac{1}{2}$

It is false.

∴ I am not agree with Rakesh solution

v) $3x^{-1} = \frac{1}{3x}$

$3 \times 3 = \frac{x}{x}$

$9 = x^0$

$9 = 1 \quad [\because x^0 = 1]$

It is False

∴ I am not agree with Rakesh's Solution.