Case Study - 3

We know from our study of circular motion that the magnitude of linear velocity v of a particle moving in a circle is related to the angular velocity of the particle ω by the simple relation u = ω r , where r is the radius of the circle.
In vector form $v = \omega \times r$. We observe that at any given instant the relation v =ω r applies to all particles of the rigid body.
In rotational motion the concept of angular acceleration in analogy with linear acceleration defined as the time rate of change of velocity in translational motion.
We define angular acceleration α as the time rate of change of angular velocity;
Thus,

$$\alpha = \frac{d\omega}{dt}.$$

The rotational analogue of force in linear motion is moment of force. It is also referred to as torque or couple. (We shall use the words moment of force and torque interchangeably.)
If a force acts on a single particle at a point P whose position with respect to the origin O is given by the position vector r. the moment of the force acting on the particle with respect to the origin O is defined as the vector product

$\tau = r \times F$

The moment of force (or torque) is a vector quantity. The symbol τ stands for the Greek
Letter tau. The magnitude of t is

$\tau = r F \sin\theta$

Where r is the magnitude of the position vector r, i.e. the length OP, F is the magnitude of force F and θ is the angle between r and F.

1. **Torque is analogues to**
   a. Force
   b. Velocity
   c. Acceleration
   d. None of these

2. **Which of the following relation is correct**
   a. $v = r \times \omega$
   b. $v = \omega \times r$
   c. $\omega = v \times r$
   d. None of these

3. **Write a note on relation between angular velocity and linear velocity.**
4. **Define angular acceleration.**
5. **Define torque.**

Answer key - 3

1. A

2. b

3. The magnitude of linear velocity v of a particle moving in a circle is related to the angular velocity of the particle ω by the simple relation u = ω r , where r is the radius of the circle.
   In vector form v = ω × r.

4. We define angular acceleration α as the time rate of change of angular velocity; Thus,
   $\alpha = \frac{d\omega}{dt}$.

5. The rotational analogue of force in linear motion is moment of force. It is also referred to as torque or couple.
   If a force acts on a single particle at a point P whose position with respect to the origin O is given by the position vector r. the moment of the force acting on the particle with respect to the origin O is defined as the vector product
   $\tau = r \times F$
   $\tau = r\,F \sin\theta$.
   Where r is the magnitude of the position vector r, F is the magnitude of force F and θ is the angle between r and F.

Case Study - 4

Just as the moment of a force is the rotational analogue of force in linear motion, the quantity angular momentum is the rotational analogue of linear momentum. Like moment of a force, angular momentum is also a vector product. It could also be referred to as moment of (linear) momentum. From this term one could guess how angular momentum is defined. Consider a particle of mass m and linear momentum p at a position r relative to the origin O. The angular momentum l of the particle with respect to the origin O is defined to be

$L = r \times p$

The magnitude of the angular momentum vector is

$L = r\, p \sin\theta$

Where p is the magnitude of p and θ is the angle between r and p.

The time rate of change of the angular momentum of a particle is equal to the torque acting on it and given by

$\tau = dL/dt$

This is the rotational analogue of the equation F = dp/dt, which expresses Newton's second law for the translational motion of a single particle.

If $\tau_{ext} = 0$ then $dL/dt = 0$ or L = constant.

Thus, if the total external torque on a system of particles is zero, then the total angular momentum of the system is conserved, i.e. remains constant called as law of conservation of angular momentum.

1. **Angular momentum is**
   a. Vector
   b. Scalar

2. **the total angular momentum of the system is conserved if**
   a. total external force become zero
   b. total external torque become zero
   c. total linear momentum become zero
   d. None of these

3. **Give statement for conservation of angular momentum.**
4. **Define torque in terms of angular momentum**
5. **Define angular momentum**

Answer key - 4

1. A

2. b

3. If the total external torque on a system of particles is zero, then the total angular momentum of the system is conserved, i.e. remains constant called

as law of conservation of angular momentum. If $\tau_{ext} = 0$ then $dL/dt = 0$ or $L =$ constant.

4. The time rate of change of the angular momentum of a particle is equal to the torque acting on it and given by
   $\tau = dL/dt$

5. The quantity angular momentum is the rotational analogue of linear momentum. The angular momentum l of the particle with respect to the origin O is defined to be
   $L = r \times p$
   The magnitude of the angular momentum vector is
   $L = r\, p \sin\theta$
   Where p is the magnitude of p and θ is the angle between r and p.s

Case Study - 5

The kinetic energy of a rolling body can be separated into kinetic energy of translation and kinetic energy of rotation. This is a special case of a general result for a system of particles, according to which the kinetic energy of a system of particles (K) can be separated into the kinetic energy of translational motion of the centre of mass ($MV^2/2$) and kinetic energy of rotational motion about the centre of mass of the system of particles (*K*’). Thus

$$\boldsymbol{K = K' + MV^2/2}$$
$$\boldsymbol{K = \frac{1}{2} mv^2 \,(1+k^2/R^2)}$$

This applies to any rolling body: a disc, a cylinder, a ring or a sphere.
Power in rotational motion: expression for power in rotational motion can be obtained by analogy from linear motion. Power in the case of linear motion is given by
P = Force*velocity = Fv.
Therefore power in rotational motion is given by,
$P = \tau\omega$
Where τ is torque and ω is angular velocity.

1. Quantity in rotational motion analogous to force in linear motion is

a. Torque
b. Velocity
c. Power
d. None of these

2. **Kinetic energy in rolling motion is the sum of**
    a. KE in translation + KE in rotation
    b. KE in translation + KE in linear
    c. KE in gravitation + KE in rotation
    d. None of these

3. **In Expression for power in rotational motion is product of**
    a. Torque and angular velocity
    b. Force and angular velocity
    c. Mass and velocity
    d. None of these

4. **What is kinetic energy in rolling motion? Give its formula.**
5. **Write a note on power in rotational motion.**

Answer key - 5

1. a

2. a

3. a

4. The kinetic energy of a rolling body can be separated into kinetic energy of translation and kinetic energy of rotation. This is a special case of a general result for a system of particles, according to which the kinetic energy of a system of particles (K) can be separated into the kinetic energy of translational motion of the centre of mass ($MV^2/2$) and kinetic energy of rotational motion about the centre of mass of the system of particles (*K*'). Thus
    $\boldsymbol{K = K' + MV^2/2}$
    $\boldsymbol{K = \frac{1}{2} mv^2 \, (1+k^2/R^2)}$
    This applies to any rolling body: a disc, a cylinder, a ring or a sphere.

5. Expression for power in rotational motion can be obtained by analogy from linear motion. Power in the case of linear motion is given by
   P = Force*velocity = Fv.
   Therefore power in rotational motion is given by,
   $P = \tau\omega$
   Where τ is torque and ω is angular velocity.