Case Study - 4

Simple harmonic motion

Simple harmonic motion is oscillatory motion in which body performs oscillatory motion about mean position and the restoring force is directly proportional to displacement from mean position and always directed towards mean position.

Simple harmonic motion and uniform circular motion

In this section, we show that the projection of uniform circular motion on a diameter of the circle follows simple harmonic motion. A simple experiment can help us visualize this connection. Tie a ball to the end of a string and make it move in a horizontal plane about a fixed point with a constant angular speed. The ball would then perform a uniform circular motion in the horizontal plane. Observe the ball sideways or from the front, fixing your attention in the plane of motion. The ball will appear to execute to and fro motion along a horizontal line with the point of rotation as the midpoint. You could alternatively observe the shadow of the ball on a wall which is perpendicular to the plane of the circle. In this process what we are observing is the motion of the ball on a diameter of the circle normal to the direction of viewing.

Hence it conclude that if the particle performs uniform circular motion then its projection on any of the diameter performs simple harmonic motion with same time period.

Velocity and acceleration in SHM

The speed of a particle v in uniform circular motion is its angular speed ω times the radius of the circle A. $V = \omega A$

Where A is amplitude of SHM and ω is angular velocity

For simplicity, let us put $\phi = 0$ and write the expression for x (t), v (t) and

$x(t) = A \cos \omega t$, $v(t) = -\omega A \sin \omega t$,

acceleration in SHM is given by

$a(t) = -\omega^2 A \cos \omega t$

force in SHM

Using Newton's second law of motion, and the expression for acceleration of a particle undergoing SHM the force acting on a particle of mass m in SHM is

$F(t) = ma$

$= -m\omega^2 x(t)$

$= -K x(t)$

Where $k = m\omega^2$ Or $\omega = \sqrt{\frac{K}{m}}$

Like acceleration, force is always directed towards the mean position—hence it is sometimes called the restoring force in SHM.

1. **In SHM magnitude of velocity is given by**
   a. ωA
   b. $\omega^2 A$
   c. ωA^2
   d. none of these

2. **In SHM magnitude of acceleration is given by**
   a. ωA
   b. $\omega^2 A$
   c. ωA^2
   d. none of these

3. **define Simple harmonic motion**

4. **give relation between Simple harmonic motion and uniform circular motion**

5. **give expressions for velocity ,acceleration and force in SHM with initial phase zero**

Answer key – 4

1. a

2. b

3. Simple harmonic motion is oscillatory motion in which body performs oscillatory motion about mean position and the restoring force is directly proportional to displacement from mean position and always directed towards mean position.

4. Hence it conclude that if the particle performs uniform circular motion then its projection on any of the diameter performs simple harmonic motion with same time period.

5. Velocity and acceleration in SHM

The speed of a particle v in uniform circular motion is its angular speed ω times theradius of the circle A. V = ω A

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Case Study - 5

ENERGY IN SIMPLE HARMONIC MOTION

Both kinetic and potential energies of a particle in SHM vary between zero and their maximum values. We have seen that the velocity of a particle executing SHM, is a periodic function of time. It is zero at the extreme positions of displacement. Therefore, the kinetic energy (K) of such a particle, which is defined as

$K = (½) mv^2$

$K = (1/2) m\omega^2 A^2 \sin^2 (\omega t + \phi)$

$K = (1/2) kA^2 \sin^2 (\omega t + \phi)$

Is also a periodic function of time, being zero when the displacement is maximum and maximum when the particle is at the mean position.

What is the potential energy (U) of a particle executing simple harmonic motion? we have seen that the concept of potential energy is possible only for conservative forces. The spring force

$F = -kx$ is a conservative force, with associated potential energy

$U = (1/2) kx^2$

Hence the potential energy of a particle executing simple harmonic motion is,
$U = (1/2)\ kx^2$
$U = (1/2)\ kA^2\cos^2(\omega t + \phi)$
Thus, the potential energy of a particle executing simple harmonic motion is also periodic, with period T/2, being zero at the mean position and maximum at the extreme displacements.
The total energy, E, of the system is,
$E = U + K$
$E = (1/2)\ kA^2\sin^2(\omega t + \phi) + (1/2)\ kA^2\cos^2(\omega t + \phi)$
Using the familiar trigonometric identity,
$E = (1/2)\ kA^2$
The total mechanical energy of a harmonic oscillator is thus independent of time as expected for motion under any conservative force. Answer the following

1. **At mean position in SHM kinetic energy is**
   a. Maximum
   b. Minimum
   c. Zero
   d. None of this

2. **At mean position in SHM potential energy is**
   a. Maximum
   b. Minimum
   c. Infinite
   d. None of these

3. **Derive expression for kinetic energy in SHM.**

4. **Derive expression for potential energy in SHM.**

5. **Derive expression for total energy in SHM.**

Answer key-5

1. A

2. B

3. , the kinetic energy (K) of particle performing SHM with speed V and mass M, which is defined as
   $K = (½)\ mv^2$
   $K = (1/2)\ m\omega^2 A^2 \sin^2 (\omega t + \phi)$
   $K = (1/2)\ kA^2 \sin^2 (\omega t + \phi)$
   Is also a periodic function of time, being zero when the displacement is maximum and maximum when the particle is at the mean position.

4. The spring force is given by
   $F = -kx$ is a conservative force, with associated potential energy
   $U = (1/2)\ kx^2$
   Hence the potential energy of a particle executing simple harmonic motion is,
   $U = (1/2)\ kx^2$
   $U = (1/2)\ kA^2 \cos^2 (\omega t + \phi)$
   Thus, the potential energy of a particle executing simple harmonic motion is also periodic, with period T/2, being zero at the mean position and maximum at the extreme displacements.

5. The total energy, E, of the system is,
   $E = U + K$
   $E = (1/2)\ kA^2 \sin^2 (\omega t + \phi) + (1/2)\ kA^2 \cos^2 (\omega t + \phi)$
   Using the familiar trigonometric identity,
   $E = (1/2)\ kA^2$
   The total mechanical energy of a harmonic oscillator is thus independent of time as expected for motion under any conservative force.