Case 4

We should learn how errors combine in various mathematical operations. For this, we use the following procedure.

Error of a sum or a difference: Suppose two physical quantities A and B havemeasured values$A \pm \Delta A$, $B \pm \Delta B$ respectively whereΔA andΔB are their absolute errors. Wewish to find the errorΔZ in the sum$Z = A + B$.We have by addition,

$Z \pm \Delta Z = (A \pm \Delta A) + (B \pm \Delta B)$.

The maximum possible error in Z

$\Delta Z = \Delta A + \Delta B$

For the difference$Z = A - B$, we have

$Z \pm \Delta Z = (A \pm \Delta A) - (B \pm \Delta B)$

$= (A - B) \pm \Delta A \pm \Delta B$

or, $\pm \Delta Z = \pm \Delta A \pm \Delta B$ The maximum value of the error ΔZ is again$\Delta A + \Delta B$.

Hence the rule: When two quantities are added or subtracted, the absolute error in the final result is the sum of the absolute errors in the individual quantities.

Error of a product or a quotient

Suppose $Z = AB$ and the measured values of A and B are $A \pm \Delta\Delta A$ and $B \pm \Delta B$. Then

$Z \pm \Delta Z = (A \pm \Delta A)(B \pm \Delta B)$

$= AB \pm B\,\Delta A \pm A\,\Delta B \pm \Delta A\,\Delta B$.

Dividing LHS by Z and RHS by AB we have,

$1 \pm (\Delta Z/Z) = 1 \pm (\Delta A/A) \pm (\Delta B/B) \pm (\Delta A/A)(\Delta B/B)$.

Since ΔA and ΔB are small, we shall ignore their product.

Hence the maximum relative error

$\Delta Z/ Z = (\Delta A/A) + (\Delta B/B)$.

You can easily verify that this is true for division also.

Hence the rule : When two quantities are multiplied or divided, the relative error in the result is the sum of the relative errors in the multipliers.

Error in case of a measured quantity raised to a power

if $Z = A^p B^q/C^r$ then

$\Delta Z/Z = p\,(\Delta A/A) + q\,(\Delta B/B) + r\,(\Delta C/C)$.

Hence the rule: The relative error in a physical quantity raised to the power k is the k times the relative error in the individual quantity. Answer the following.

1. In the sum$Z = A + B$ the maximum value of the error ΔZ is
   a. $\Delta A + \Delta B$
   b. $\Delta A - \Delta B$.
   c. $\Delta A\,\Delta B$.
   d. None of the above
2. In the sum$Z = A - B$ the maximum value of the error ΔZ is
   a. $\Delta A + \Delta B$
   b. $\Delta A - \Delta B$.
   c. $\Delta A\,\Delta B$.
   d. None of the above
3. Write down expression for absolute error when two quantities are added or subtracted

4. Write down expression for absolute error when two quantities are multiplied or divided
5. Write down expression for absolute error in case of measured quantity raised to a power

Answer key-

1. a
2. a
3. **Error of a sum or a difference:** Suppose two physical quantities A and B havemeasured values$A \pm \Delta A$, $B \pm \Delta B$ respectively whereΔA andΔB are their absolute errors. Wewish to find the errorΔZ in the sum$Z = A + B$.We have by addition,

   $Z \pm \Delta Z = (A \pm \Delta A) + (B \pm \Delta B)$.

   The maximum possible error in Z

   $\Delta Z = \Delta A + \Delta B$

   For the difference$Z = A - B$, we have

   $Z \pm \Delta Z = (A \pm \Delta A) - (B \pm \Delta B)$

   $= (A - B) \pm \Delta A \pm \Delta B$

   or, $\pm \Delta Z = \pm \Delta A \pm \Delta B$ The maximum value of the error ΔZ is again$\Delta A + \Delta B$.

   Hence the rule : When two quantities are added or subtracted, the absolute error in the final result is the sum of the absolute errors in the individual quantities.
4. **Error of a product or a quotient**

   Suppose Z = AB and the measured values of A and B are $A \pm \Delta\Delta A$ and $B \pm \Delta B$. Then

   $Z \pm \Delta Z = (A \pm \Delta A)(B \pm \Delta B)$

   $= AB \pm B\,\Delta A \pm A\,\Delta B \pm \Delta A\,\Delta B$.

   Dividing LHS by Z and RHS by AB we have,

   $1 \pm (\Delta Z/Z) = 1 \pm (\Delta A/A) \pm (\Delta B/B) \pm (\Delta A/A)(\Delta B/B)$.

   Since ΔA and ΔB are small, we shall ignore their product.

   Hence the maximum relative error

   $\Delta Z/ Z = (\Delta A/A) + (\Delta B/B)$.

   You can easily verify that this is true for division also.

   Hence the rule: When two quantities are multiplied or divided, the relative error in the result is the sum of the relative errors in the multipliers.
5. **Error in case of a measured quantity raised to a power**

If $Z = A^p B^q/C^r$ then

$\Delta Z/Z = p\,(\Delta A/A) + q\,(\Delta B/B) + r\,(\Delta C/C)$.

Hence the rule: The relative error in a physical quantity raised to the power k is the k times the relative error in the individual quantity

Case 5

Absolute Error, Relative Error and Percentage Error

a. Suppose the values obtained in several measurements are a1, a2, a3...., an. The arithmetic mean of these values is taken as the best possible value of the quantity under the given conditions of measurement as:

$a_{mean} = (a1+a2+a3+...+an) / n$.

The magnitude of the difference between the individual measurement and the true value of the quantity is called the absolute error of the measurement. This is denoted by $|\Delta a|$. In absence of any other method of knowing true value, we considered arithmetic mean as the true value. Then the errors in the individual measurement values from the true value are

$\Delta a_1 = a_1 - a_{mean}$
$\Delta a_2 = a_2 - a_{mean}$
$\Delta a_n = a_n - a_{mean}$

The Δa calculated above may be positive in certain cases and negative in some other cases. But absolute error $|\Delta a|$ will always be positive.

b. The arithmetic mean of all the absolute errors is taken as the final or mean absolute error of the value of the physical quantity a. It is represented by Δa_{mean}.Thus,

$\Delta a\text{mean} = (|\Delta a_1|+|\Delta a_2|+|\Delta a_3|+...+|\Delta a_n|)/n$

c. Instead of the absolute error, we often use the relative error or the percentage error (δa). The relative error is the ratio of the mean absolute error Δamean to the mean value a_{mean} of the quantity measured

Relative error $= \frac{\Delta a\text{mean}}{a\text{mean}}$

When the relative error is expressed in per cent, it is called the percentage error (δa). Thus, Percentage error

$\delta a = (\Delta a_{mean} / a_{mean}) \times 100\%$

1. Absolute error $|\Delta a|$ will
   a. Always be negative
   b. Always be positive
   c. Can be positive or negative
   d. None of these
2. $\frac{\Delta a\text{mean}}{a\text{mean}}$ is the
   a. Absolute error
   b. Relative error
   c. Percentage error
   d. None of these
3. What is absolute error
4. What is relative error
5. What is the percentage error

Answer key-

1. b
2. b
3. Suppose the values obtained in several measurements are a1, a2, a3...., an. The arithmetic mean of these values is taken as the best possible value of the quantity under the given conditions of measurement as:

   $a_{mean} = (a1+a2+a3+...+an) / n$.

   The magnitude of the difference between the individual measurement and the true value of the quantity is called the absolute error of the measurement. This is denoted by $|\Delta a|$. In absence of any other method of knowing true value, we considered arithmetic mean as the true value. Then the errors in the individual measurement values from the true value are

$$\Delta a_1 = a_1 - a_{mean}$$

$$\Delta a_2 = a_2 - a_{mean}$$

$$\Delta a_n = a_n - a_{mean}$$

The Δa calculated above may be positive in certain cases and negative in some other cases. But absolute error $|\Delta a|$ will always be positive.

4. The relative error is the ratio of the mean absolute error Δamean to the mean value a_{mean} of the quantity measured

   Relative error $= \frac{\Delta a\text{mean}}{a\text{mean}}$

5. When the relative error is expressed in per cent, it is called the percentage error (δa). Thus, Percentage error is given by

   $\delta a = (\Delta a_{mean} / a_{mean}) \times 100\%$